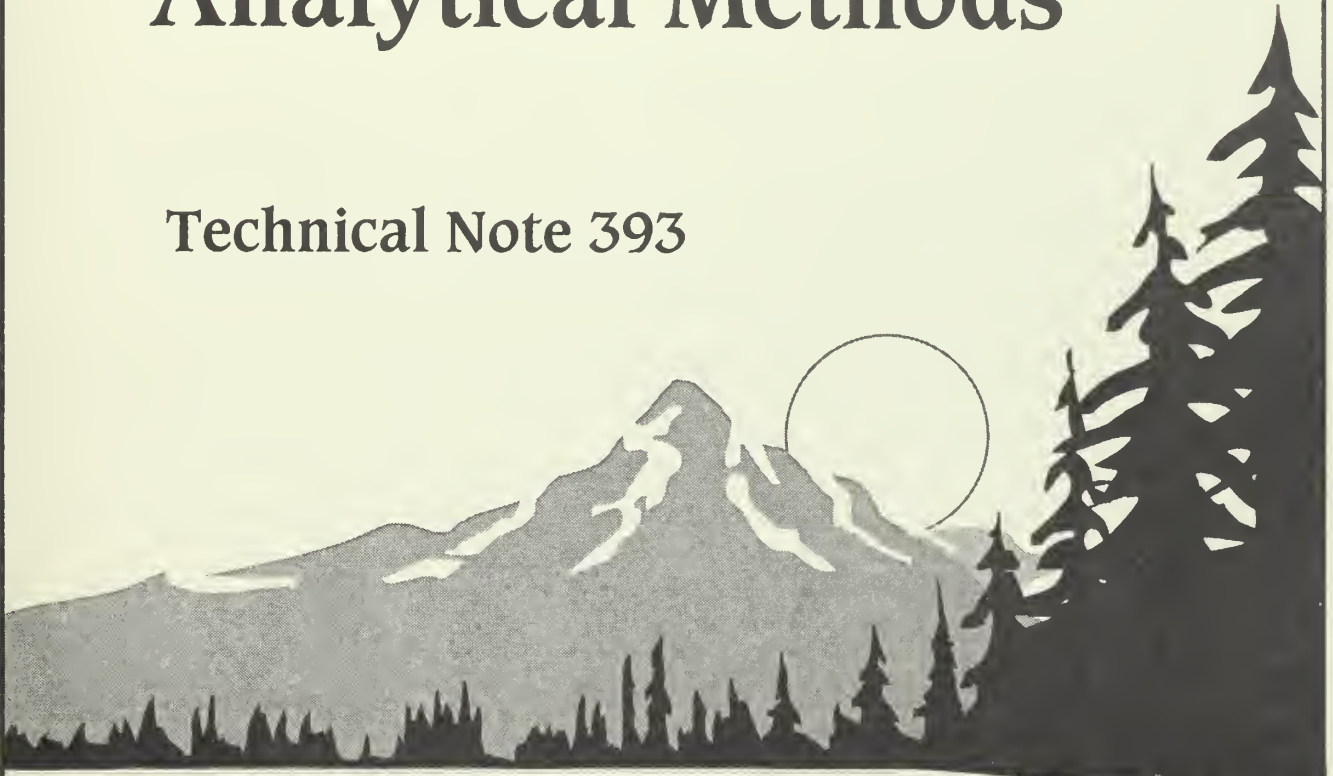# Geohydrology : Analytical Methods

Technical Note 393

U. S. DEPARTMENT OF THE INTERIOR
Bureau of Land Management

September 1995

Table for Flow Conversion

Unit	m³/sec	m³/day	ℓ/sec	ft³/sec	ft³/day	ac-ft/day	gal/min	gal/day	mgd
1 cubic meter/ second	1	9.64×10^4	10^3	35.31	3.051×10^6	70.05	1.58×10^4	2.282×10^7	22.824
1 cubic meter/ day	1.157×10^{-5}	1	0.0116	4.09×10^{-4}	35.31	8.1×10^{-4}	0.1835	264.17	2.64×10^{-4}
1 liter/ second	0.001	86.4	1	0.0353	3051.2	0.070	15.85	2.28×10^4	2.28×10^{-2}
1 cu. foot/ second	0.0283	2446.6	28.32	1	8.64×10^4	1.984	448.8	6.46×10^5	0.646
1 cu. foot/ day	3.28×10^7	0.02832	3.28×10^{-4}	1.16×10^{-5}	1	2.3×10^{-5}	5.19×10^{-3}	7.48	7.48×10^{-6}
1 acre-foot/ day	0.0143	1233.5	14.276	0.5042	43,560	1	226.28	3.259×10^5	0.3258
1 gallon/ minute	6.3×10^5	5.451	0.0631	2.23×10^{-3}	192.5	4.42×10^{-3}	1	1440	1.44×10^{-3}
1 gallon/ day	4.3×10^8	3.79×10^{-3}	4.382×10^{-5}	1.55×10^{-6}	0.11337	3.07×10^{-6}	6.94×10^{-4}	1	10^{-6}
1 million gallons/ day	4.38×10^{-2}	3785	43.82	1.55	1.337×10^5	3.07	694	10^6	1

Conversion Values for Hydraulic Conductivity

1 gal/day/ft²	=	0.0408 m/day
1 gal/day/ft²	=	0.134 ft/day
1 gal/day/ft²	=	4.72×10^{-5} cm/sec
1 ft/day	=	0.305 m/day
1 ft/day	=	7.48 gal/day/ft²
1 ft/day	=	3.53×10^{-4} cm/sec
1 cm/sec	=	864 m/day
1 cm/sec	=	2835 ft/day
1 cm/sec	=	21,200 gal/day/ft²
1 m/day	=	24.5 gal/day/ft²
1 m/day	=	3.28 ft/day
1 m/day	=	0.00116 cm/sec

Geohydrology: Analytical Methods

Technical Note 393

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Abstract

This guide analyzes the field methods involved in conducting a geohydrologic analysis, including pretest water level monitoring, pumping phase, and recovery phase. Selected methods of analytical analysis are reviewed with reference to the geohydrologic setting, the stress placed on the aquifer by the pumping well, the observation of aquifer response, the mathematical solution to the hydraulic head response in the aquifer, and the technique for calculating the hydraulic properties of the aquifer. Type curves are included for selected aquifer test methods.

Acknowledgements

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Introduction

Purpose and Scope

The need for a comprehensive geohydrologic analytical guide for Bureau of Land Management field offices became apparent as geohydrologists and water resource specialists were called upon to interpret and evaluate data in support of ground water resource projects, with specific emphasis on mine dewatering projects. Today's mining operations take place, for the most part, in the form of open pit and underground workings, all of which require, to some degree, dewatering of geological materials for mineral extraction and for safety. This is particularly important because of the increasing emphasis placed on water resources nationwide. The purpose of this guide is to provide methods for geohydrologic analysis as applied by geohydrologists and water resource specialists working on mine dewatering and other water resource projects.

The guide presents analyses of a variety of ground water problems encountered in the planning and development of Environmental Assessments (EAs) and Environmental Impact Statements (EISs) for mine dewatering projects and other water resource projects. These problems include analysis of depletions caused by pumping, estimated seepage, analysis of drawdown, and estimates of permeabilities for hydrostratigraphic units. In analyzing these and other ground water problems, theoretical assumptions and limitations are outlined and specific methods are addressed through the use of tables, figures, and solution equations.

Previous Work

An extensive *Summary of Hydraulic Test Methods* is presented in Appendix A of this work. Of the original papers describing hydraulic test methods, the paper by Theis (1935) is highly recommended reading for the interested geohydrologist or water resource specialist. Theis' (1935) paper introduced the most useful method of aquifer flow hydraulics and aquifer flow concepts. In addition to these original papers, the geohydrologist or water resource specialist will find the *Selected References* section useful in providing guidance on selection of aquifer test methods, interpretation of aquifer test data, and examples of applications of hydraulic test methods. The report by Stallman (1971) is useful for the practical application of aquifer test planning and data interpretation. The report by Lohman (1972) is an extremely good text on the basic

principles of ground water hydraulics and methods with examples of their application. Reed (1980) presents the most complete collection of tables and types of curves for application of aquifer test methods to confined aquifer problems together with discussions of the analytical solutions and their limitations and applications. Other useful references include Ferris and others (1962), Walton (1962), Bentall (1962a), Hantush (1964a), Kruseman and DeRidder (1991), Dawson and Istok (1991), Fetter (1994), and Vukovic and Soro (1991). Walton (1962) gives many examples of aquifer tests including information on the geohydrologic setting, test data, and type curve applications. These references from the early 1960s are outstanding treatments of many useful hydraulic test methods. In addition, several important methods have been developed in recent years, such as methods for unconfined aquifers, pumping well storage, inertial effects, advances in slug test procedures, and solutions to boundary value problems by numerical inversion techniques (Moench and Ogata, 1984).

Geohydrologic Analytical Procedures

Geohydrologic Characteristics Determined From Aquifer Tests

An aquifer test is a controlled in situ experiment made to determine the geohydrologic characteristics of water flow and associated rocks. The test is made by measuring ground water flow or head that is produced by known hydraulic boundary conditions such as pumping wells, recharging wells, variations in head along a connected stream, or changes in weight imposed on the land surface.

The geohydrologic characteristics that can be determined from an aquifer test depend on the onsite test conditions and installations. The most common geohydrologic parameters determined are the coefficients of transmissivity, T, and storage, S, or storativity. Transmissivity is a measure of the ease in which the full thickness of the aquifer transmits water; the hydraulic conductivity, K, is a measure of the ease with which a unit thickness of the aquifer transmits water.* Therefore,

$$K = \frac{T}{b} \tag{1}$$

where b is the thickness of the aquifer. The evaluation of rates of ground water flow in an aquifer requires knowledge of hydraulic conductivity and effective porosity. Effective porosity, or drainable porosity, is a measure of the interconnected void space of a medium. The storage coefficient of an unconfined aquifer is approximately equal to the effective porosity, n. The storage coefficient of a confined aquifer is typically much smaller than that of an unconfined aquifer. Whereas water yielded to a well from an unconfined aquifer is derived principally from drainage of water from voids, water yielded to a well from a confined aquifer is derived principally by compression of the aquifer and expansion of the water. Values of effective porosity of granular materials usually range from 0.1 to 0.4; storage coefficients of confined aquifers usually range from 10^{-4} to 10^{-6}.

The relation of flow in an aquifer to the hydraulic conductivity and the hydraulic gradient are expressed by a general form of Darcy's law:

$$v = -K \frac{dh}{dl} = \frac{Q}{A} \tag{2}$$

For the definition of the symbols used in a specific equation, the reader is referred to Appendix B, List of Nomenclature and Symbols.

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where v is the flux specific discharge, also called Darcy's velocity, dh/dl is the hydraulic gradient, Q is the discharge, and A is the cross-sectional area. From these parameters of the aquifer, the rate of advective transport of a solute can be calculated by the following relation:

$$\overline{v} = (\frac{K}{n})\ (\frac{dh}{dl}) \tag{3}$$

Thus, aquifer tests do not provide a direct analysis of the parameters K and n. But, K can be determined from an aquifer test where the saturated thickness is known. The effective porosity can be estimated as the storage coefficient from tests of an unconfined aquifer. The determination of storage coefficient from an aquifer test requires analysis of the drawdown response in observation wells rather than in the pumping well. Drawdown response solely in the pumping well can be used to calculate transmissivity, but is not reliable for determination of the storage coefficient because the effective radius of the pumping well is not known (after Bedinger and others, 1988).

Application of Hydraulic Tests for Geohydrologic Systems

Aquifer tests were originally applied to wells completed in aquifers that were used for water supply. The first tests were designed simply to define the gross hydraulic properties of the water-yielding material. The earliest application of aquifer tests was in the design of well fields and in the prediction of the performance of an aquifer as a source of water supply. Aquifer test methodology has increased tremendously in sophistication as a result of more complex techniques applied to analyzing simple aquifer and boundary conditions. The type of aquifer test methods available today can provide more detailed information on the confining beds as well as flow system characteristics. Aquifer test methods can provide much more of the detail needed for characterization and analysis of hydrologic systems.

Definition of hydraulic properties is an essential element in characterization of geohydrologic systems and in design of ground water field programs for mine dewatering and water resource studies. Aquifer test methods are specifically designed to provide analysis of hydraulic properties under a certain set of geohydrologic conditions. Therefore, aquifer tests can be designed and performed to provide the type of information on the flow system that is best suited for the geohydrologic setting and the application for which the data are needed. As discussed by Bedinger and others (1988), aquifer tests can be chosen to provide information on the gross hydraulic properties of a large volume of an aquifer, the hydraulic conductivity of a relatively thin bed in the flow system, the

relative horizontal to vertical hydraulic conductivity of an aquifer, areal anisotropy of the aquifer, or the leakage from a confining bed.

The aquifer test method chosen must provide the type of information required for a given application. For example, pit and underground mine operational monitoring and mitigation programs might be enhanced by information that includes horizontal and vertical hydraulic conductivity, estimation of the rate and direction of ground water flow, spatial distribution, and the hydraulic characteristics of a specific hydrostratigraphic unit. In addition, the design of a plan for ground water reinjection or infiltration for mine operational water management might require one or more long term aquifer tests with many observation wells.

Aquifer tests require information on the geohydrology of the area and a network of one or more wells that are constructed and instrumented to provide the data necessary for analysis by the aquifer test method chosen. Unless the test area has been defined by investigations such as borings, geophysical logging, coring, surface water surveys, water level measurements, or other means, the most appropriate aquifer test method may not be chosen. Aquifer tests designed for analysis of specific hydraulic properties generally have specific requirements for layout and construction of the pumping and observation wells.

Hydraulic Test Planning, Design, and Implementation

An outline of the steps involved in the planning, design, and implementation of an aquifer test is given in the following sections.

Evaluation of the Geohydrologic System

Through an evaluation of the geohydrology of a water resources project, a conceptual model of the flow system is made. The evaluation needs to provide a concept of the nature of the aquifer's transmissivity, homogeneity, and isotropy, and whether the aquifer is confined or unconfined and, if confined, whether the aquifer is overlain or underlain by leaky or nonleaky confining beds. Emphasis is placed on the need for knowledge of the geohydrologic setting because the response of an aquifer system to stress is not unique to the geohydrology. Misunderstanding the geohydrologic setting could lead to selection of an inappropriate aquifer test method and incorrect analysis of hydraulic properties. Therefore, an accurate estimate of the flow system characteristics needs to be made, and an estimate of the hydraulic properties of the aquifer needs to be known in order to plan and implement the most representative aquifer test.

Survey of Selected Aquifer Test Methods

The available literature on aquifer test methods is extensive and each method is specific with respect to geohydrologic conditions and well control. Furthermore, each method is usually limited to a relatively simple set of aquifer characteristics and boundary conditions as opposed to the complexity of the actual area being studied. Selection of the aquifer test method as discussed by Bedinger and others (1988) is made on the basis of the geohydrology of the test site and the field test conditions. The geohydrology of the test location with regard to nonleaky confined aquifer, leaky confined aquifer, unconfined aquifer, and other natural conditions of the area determine the applicable set of aquifer test methods. The field test conditions (with regard to number and location of observation wells, if any), instrumentation for measuring water levels, screened interval, and capacity of the pump on the pumping well, determine which aquifer test methods can be applied to the data. These and other factors determine the physical constraints on stressing the aquifer and on determining the aquifer response, and may further limit the aquifer test methods applicable for analysis.

An overview of some of the more commonly used aquifer test methods and their applicability to geohydrologic conditions and field test conditions is provided in Table 1. Each test is discussed in the *Hydraulic Test Methods for Aquifers* section with information on the applicability of the methods to specific test site conditions. General guidelines for the number of observation wells and the distance of observation wells from the pumping well for the aquifer test methods are given in the next section.

Well Siting and Screened Intervals

A single well test uses the same well as the pumping well and the observation well. Many other aquifer test methods can be applied to the data from the pumping well. The applicability of the common aquifer test methods are outlined in Table 1. Determination of the transmissivity is considered representative by single-well test data, but determination of the storage coefficient is considered unreliable because of the problem of estimating the effective radius of the pumping well. Slug tests are commonly conducted in wells screened through only part of a saturated, permeable section. Tests in such wells measure the properties of only a small part of the water-yielding section. These measurements can be a benefit when information on variations in the hydraulic conductivity at many points is desired.

The distance from the observation well to the pumping well, r, will usually be discussed with reference to a distance,

Table 1. Aquifer test methods

| | Nonleaky Confined Aquifers | | | | | Leaky Confined Aquifers | Unconfined Aquifers | |
| | Isotropic | | | | Anisotropic | | | |
	Nonequilibrium Theis (1935)	Recovery Theis (1935)	Modified nonequilibrium Cooper and Jacob (1964)	Slug test Cooper and others (1967)	Radial-vertical Hantush (1966a and b; Weeks 1969)	Hantush and Jacob (1955) Hantush (1960)	Neuman (1975)	Theis (1935)
Stress on aquifer								
Constant discharge	x	x	x	—	x	x	x	x
Variable discharge	—		—	—	—	—	—	—
Instantaneous hydraulic-head change	—		—	x	—	—	—	—
Drawdown measurements								
Pumping well	x	x	x	x	—	—	—	x
Observation well	x	x	x	—	x	x	x	x
Aquifer penetration								
Pumping well:								
Full	x	x	x	x	—	x	x	x
Partial	—	—	—	—	x	—	x	x

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$$D_0 = 1.5b \sqrt{\frac{K_z}{K_r}} \tag{4}$$

where b is the aquifer thickness, K_z is the vertical hydraulic conductivity of the aquifer, and K_r is the horizontal hydraulic conductivity of the aquifer. Obviously, K_z and K_r are not known prior to a test, but they can be determined by a few tests. A general rule of thumb used by many geohydrologists where K_z/K_r is not known is to estimate D_0 as 2 or 3 times the thickness of the aquifer. For a fully penetrating pumping well, observation wells can be fully or partially penetrating and either within or without a distance $D_0 = 1.5b$ (K_z/K_r) from the pumping well. If the pumping well partially penetrates the aquifer, the observation wells can be either fully penetrating within a distance of D_0 from the pumping well or they can be partially or fully penetrating outside a distance of D_0 from the pumping well. No observation wells are used in slug tests; the slug well should be fully penetrating, but usually is partially penetrating. In applying the radial-vertical methods of Hantush (1966a and b) and Weeks (1969) to determine horizontal and vertical permeability, the pumping well needs to be partially penetrating; the observation wells need to be piezometers that are either open at a point or screened for only a short vertical distance. The piezometers need to be within the distance D_0 of the pumping well. In applying the general method of Neuman (1975) for unconfined aquifers, the fully penetrating pumping and observation wells must meet the requirements of distance D_0 from the pumping well. The family of type curves for this method is presented in the *Solution of Boulton and Neuman* section of this guide. The method of Neuman (1975) requires fully or partially penetrating wells with greater or lesser distances of D_0 from pumping wells to observation wells.

The nonequilibrium method of Theis (1935) is applicable to unconfined aquifers where the pumping well is fully penetrating, the observation well is fully or partially penetrating, and the observation well is greater than $b/r(K_z/K_r)$ from the pumping well for times greater than $10S_y r^2/T$ (Neuman, 1975, p. 337), and where S_y is the specific yield. There are two zones where this method can be applied using fully penetrating observation wells. The first zone is far from the pumping well at later times where $r > b/r(K_z/K_r)$ and $t > S_y r^2/T$ (Neuman, 1975, p. 338). The second zone is near the pumping well at early times where $r < 0.03 \, b/r(K_z/K_r)$ and $t < Sr^2/T$ (Neuman, 1975), where S is the storage coefficient.

Establishing Baseline Water Level Fluctuations

Water levels continually fluctuate in response to local or regional stresses that are imposed on the flow system, such as recharge, discharge, changes in hydraulic head at the boundaries of the flow system,

barometric changes, and weight on the land surface. The effects of these background water level fluctuations in the locality of an aquifer test ideally are small; but even so, they cannot be discounted.

Measurements made before and after the aquifer test to detect regional water level trends are required to interpret the background water level trend and to more accurately identify drawdown. The water level trend before and after an aquifer test at a theoretical observation well is shown in Figure 1. The effect of drawdown imposed by the pumping well is superposed on the background water level fluctuations. The drawdown is the distance between the background water level trend and the water level in the observation well. From the theory of ground water hydraulics, it is noted that the recovery period is longer than the pumping period. The recovery of water level after stoppage of pumping is measured from the interpreted drawdown curve.

An inverse relation between barometric pressure and change in water level in confined aquifers is commonly identified. Water levels in unconfined aquifers are unaffected by changes in barometric pressure. Water levels in confined aquifers need to be corrected for barometric changes during an aquifer test according to the barometric efficiency of the aquifer.

Step Drawdown Test

The step drawdown test is usually conducted to provide a basis for selecting the discharge rate for a long-term aquifer test. A step drawdown test is a preliminary aquifer test that uses incremental increases in the pumping rate starting from an initial slow pumping rate to successively faster pumping rates. The test usually is conducted in 1 day. Pumping times need to be the same for each rate; either the water level may be allowed to recover between pumping periods or the pumping rate may be increased without a recovery period. The duration of each step needs to be long enough (usually 1 to 2 hours is adequate) for the rate of drawdown to become virtually stable (Figure 2).

In a pumping well, the major part of the drawdown occurs in the formation where the energy provided in overcoming the frictional resistance of the formation against the slowly moving water is directly proportional to the rate of motion. Another important part of the loss is a function of the proportionality of the velocity approaching the square of the velocity. A relation between these two components of drawdown is expressed by Jacob (1947):
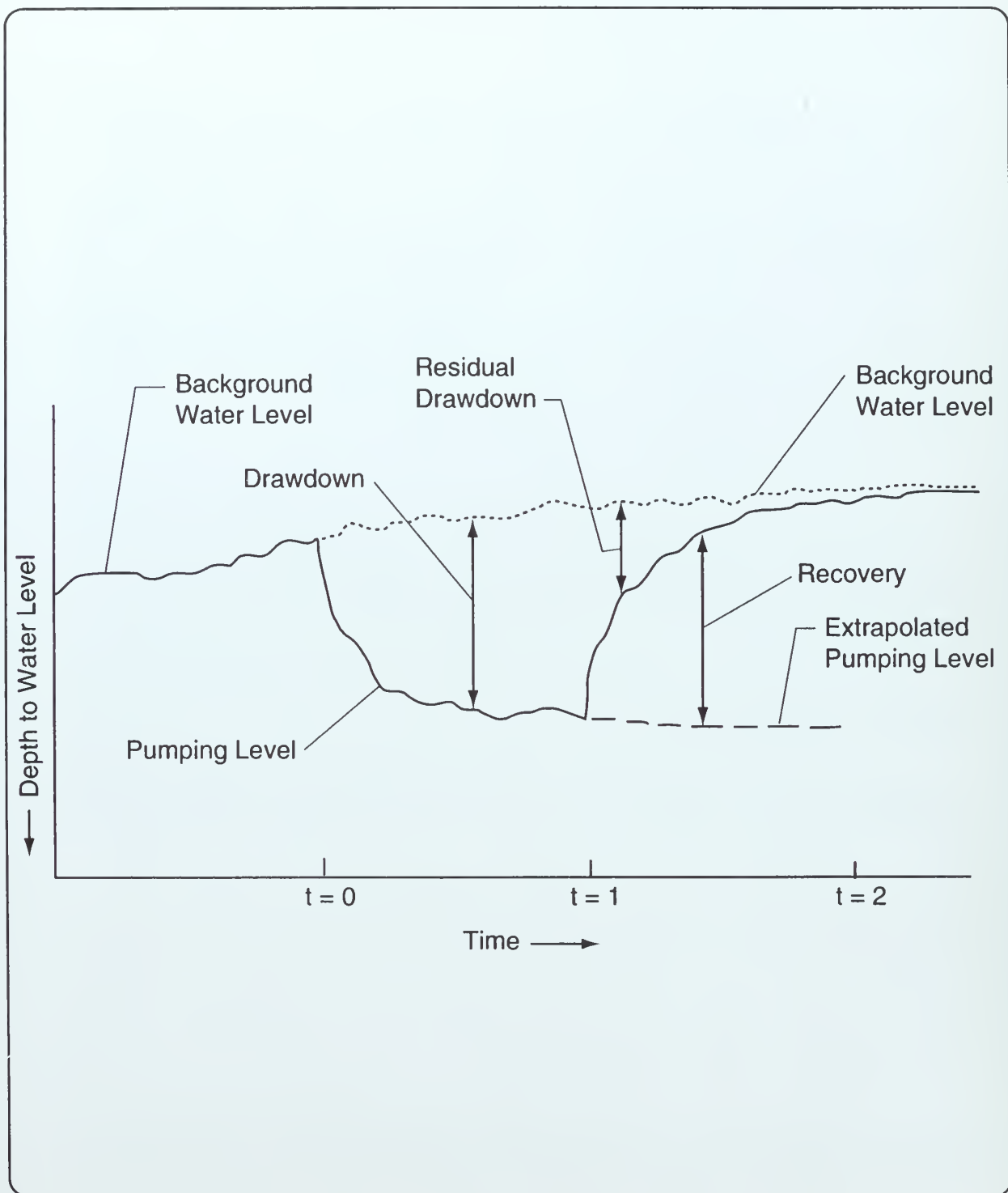
Figure 1. Hydrograph of hypothetical observation well showing background water levels, and drawdown and recovery of water level. Drawdown begins at t=0 and ends at t=1. Recovery is not complete at t=2. Residual drawdown at t=2 equals the drawdown that would have occurred from t=1 to t=2 had pumping continued at a constant rate (After Vukovic and Soro, 1991).
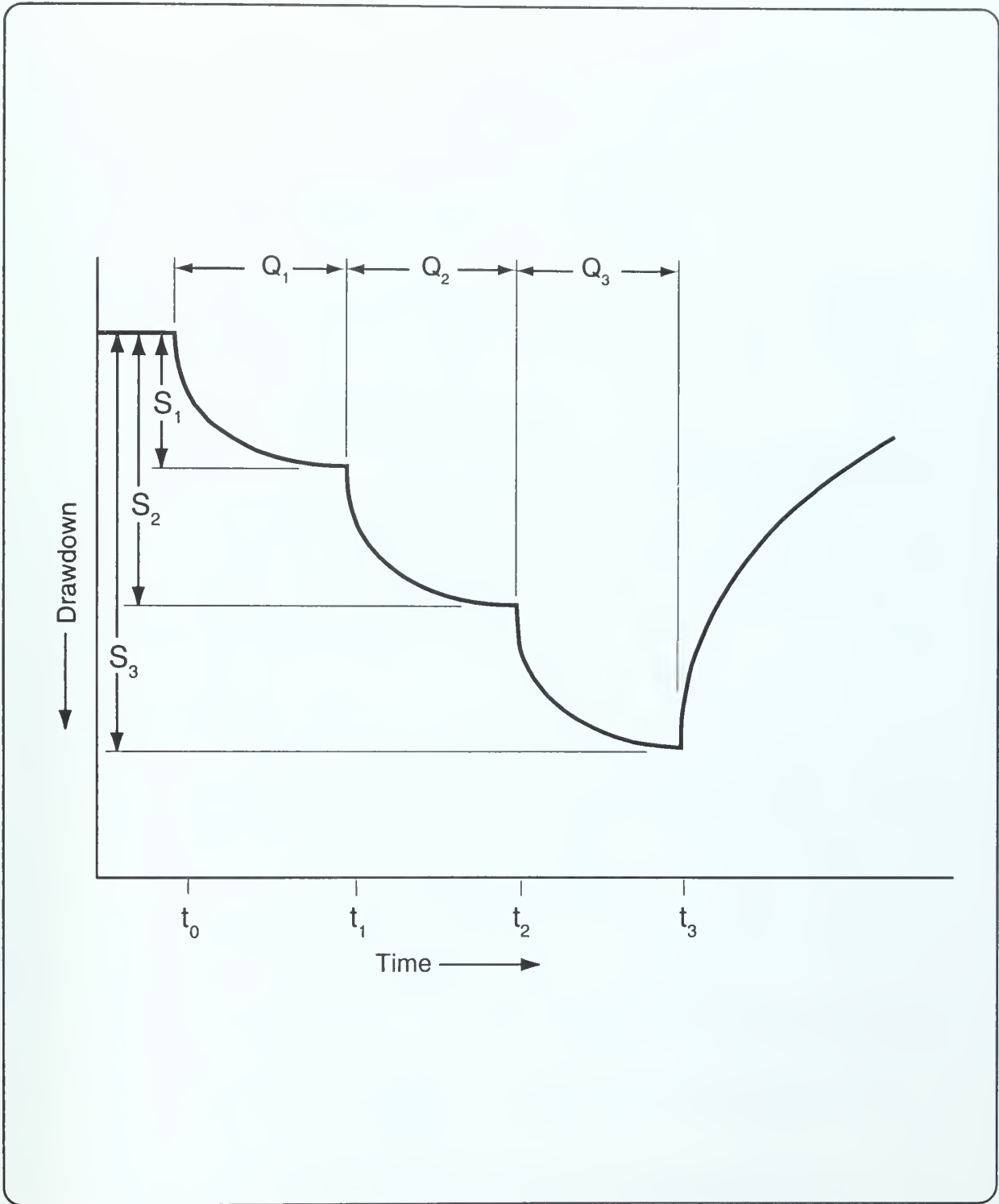
Figure 2. Hydrograph of a step-drawdown test: t is equally spaced time(s), s is drawdown, and Q is the discharge rate(s) (After Vukovic and Soro, 1991).
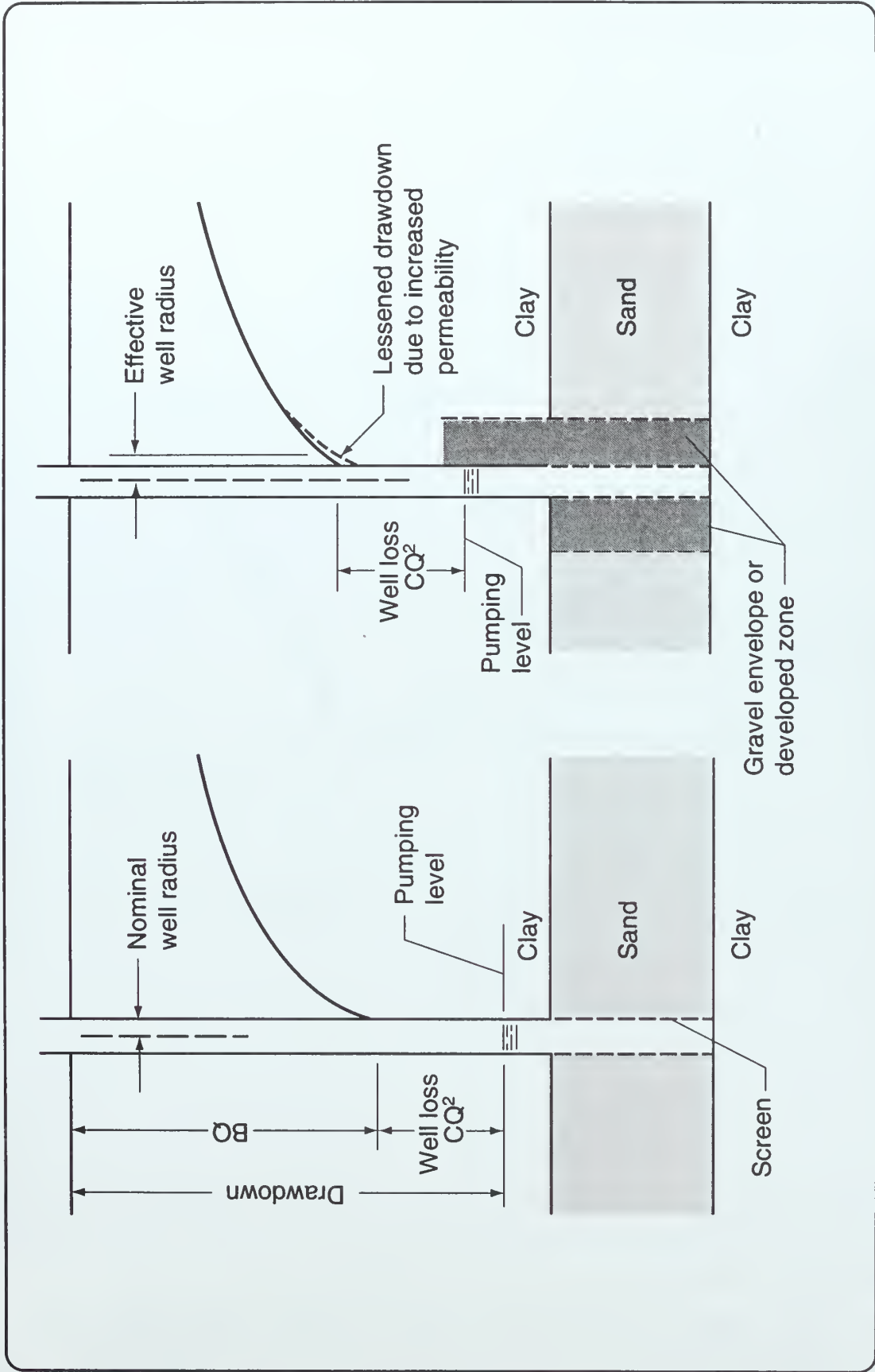
Figure 3. Cross section through a pumping well showing components of drawdown and effective well radius (After Dawson and Istok, 1991).

$$s_w = BQ + CQ^2 \tag{5}$$

where s_w is the drawdown in the pumping well, B is the coefficient of head loss linearly related to the flow, and C is the coefficient of head loss due to turbulent flow in the well, aquifer, and across the well screen (Cooley and Cunningham, 1979). Components of drawdown are shown in Figure 3. Rorabaugh (1953) presents a more general form of equation, substituting n for the exponent 2.

Equation 5 expresses the well loss component of drawdown in proportion to the square of the discharge, Q. Bierschenk (1964) presents a graphical method for determining the constants B and C in equation 5.

Developing an Aquifer Test Plan

The following guidelines for specifications and tolerances of measurements for the aquifer test are primarily from a report by Stallman (1971). For additional detail and discussion of these items, the reader is referred to Stallman (1971) and Driscoll (1986). These items may be used as a checklist that includes tasks that need to be done before, during, and after the test.

1. Pumping well needs to:

    a. Be equipped with reliable power, pump, and discharge control equipment to maintain the discharge rate during the aquifer test.

    b. Be equipped to carry discharge water away from pumping and observation wells.

    c. Be equipped to measure discharge at specific times during the aquifer test.

    d. Be equipped to measure the water level before, during, and after the aquifer test.

    e. Have a known diameter, depth, and screened interval(s).

    f. Have a screened interval(s) compatible with the aquifer test method.

    g. Be used for a step drawdown test to determine the discharge rate for the aquifer test.

2. Observation well needs to:

    a.   Be used for water level measurements during the step draw-down test to assure hydraulic connection with the aquifer, determine accuracy of water level measurements, and determine response to discharge from the pumping well.

    b.   Be a known radial distance from the pumping well.

    c.   Be used to measure baseline water levels to determine the trend of these levels before the aquifer test begins.

    d.   Have a known diameter, depth, and screened interval(s).

    e.   Have a screened interval(s) and a distance from the pumping well compatible with aquifer test methods to be used in the analysis.

3. Aquifer test method(s) need to be:

    a.   Selected for analysis based on geohydrologic condition and test area installations, especially pumping and observation wells and theorized response of flow system.

    b.   Known so that applicable type curves, graph paper, and materials for onsite analyses of data can be assembled.

4. Records of and the tolerances in measurements for the following are needed for analysis:

    a.   Pumping well discharge (±10 percent).

    b.   Depth to water in pumping and observation wells below measuring point (±0.01 ft).

    c.   Distance from pumping well to each observation well (±0.5 percent).

    d.   Synchronous time (±1 percent of time since pumping initiated).

    e.   Description of measuring points.

    f.   Elevation of measuring points (±0.01 ft).

g.   Vertical distance between measuring point and land surface (±0.1 ft).

h.   Total depth of all wells (±1 percent).

i.   Depth and length of screened interval(s) of all wells (±1 percent).

j.   Diameter, casing type, screen type, and method of construction of all wells (nominal).

k.   Location of all wells in plan either relative to land survey net or by latitude and longitude (accuracy dependent on individual need).

5.   Measurements of water level need to:

a.   Be made periodically in all wells 24 to 72 hours before the step drawdown test, continuing through recovery (*Establishing Baseline Water Level Fluctuations* section and Figure 1).

b.   Be made continually in all observation wells during the aquifer test.

c.   Be recorded with a logarithmically decreasing frequency during the aquifer test. For example, with discharge commencing at time zero, measure at 1, 1.2, 1.5, 2, 2.5, 3, 4, 5, 6, 7, and 8 minutes and at succeeding time multiples.

d.   Be made continually in all wells after stoppage of pumping to determine recovery for a period equal or longer in duration than the period of pumping.

e.   Be made periodically in all wells after complete recovery to determine baseline water levels.

6.   Measurements of barometric pressure need to:

a.   Be made continually during tests of confined aquifers, which are affected by barometric changes in water level. Measure barometric pressure during pretest through post-test water level measurement periods.

b.   Be recorded to calculate barometric efficiency of aquifer.

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Analysis of data as it is collected during the drawdown and recovery phase is helpful in assessing the progress of the aquifer test and in determining the time period necessary for the drawdown and recovery phases.

The drawdown phase of an aquifer test provides the primary data for analysis of aquifer characteristics. Activities that need to be performed during the drawdown phase are:

1. Plot measured discharge and measured depth to water for the pumping well and each observation well.

2. Correct baseline water level fluctuations and drawdown water levels for fluctuations in barometric pressure, as applicable.

3. Interpret baseline water level fluctuation from plot of corrected water level and calculate drawdown (Figure 1).

4. Plot data for analysis according to the aquifer test method or methods selected for analysis.

5. Evaluate progress of drawdown phase on the basis of analysis of the hydraulic properties by the aquifer test method or methods selected. This is done by rating the fitting of the data to type curves or rating the time at which the data plot is a straight line if using the modified nonequilibrium method.

6. Terminate drawdown phase when analyses indicate that data are adequate for calculating hydraulic properties by the aquifer test method or methods selected.

The recovery phase provides a data set for several aquifer test methods that can be used to verify the drawdown phase calculations. Recovery data analyses are considered by some geohydrologists to provide more accurate calculations of hydraulic properties. Minor variations in discharge that may have occurred during the drawdown phase are not apparent during the recovery phase. Recovery measurements in the pumping well may provide more accurate estimates of hydraulic conductivity because well loss is smaller during the later recovery phase. The recovery phase provides a transition to the baseline water levels after recovery and a basis for re-evaluating the drawdown and recovery water levels. Activities that need to be performed during the recovery phase are:

1. Continue to plot measured depth to water for the pumping well and each observation well.

2. Correct recovery water levels for fluctuations in barometric pressure, as applicable.

3. Interpret baseline water level, interpret plot of drawdown from discharge phase, and calculate recovery of water level (Figure 1).

4. Plot recovery data for analysis according to the aquifer test method selected for analysis and calculate hydraulic properties.

5. Continue recovery measurements to document post-recovery baseline water level.

After the aquifer test is completed, all data need to be reconsidered and revised analyses made as needed. The drawdowns may require revision based on final predictions of baseline water levels before and after the test. Corrections may be necessary in type curves or drawdowns for changes in discharge rate. It may become apparent that aquifer boundaries are reflected in the data and that the effects of such boundaries need to be assessed.

Type Curve Utilization

The solution to several of the principal aquifer test methods depends on the application of type curves to plots of the aquifer test data. The use of type curves is required by the existence of integral expressions in the analytical solutions that cannot be integrated directly. The application of type curves to solve for the aquifer properties follows a similar procedure in each instance. The type curve solution discussed in this guide is for the solution to the Theis (1935) nonequilibrium drawdown method. Application of type curve solutions to other methods, for example, the methods of Hantush and Jacob (1955) and Hantush (1960) for leaky confined aquifers, and the methods of Boulton (1963) and Neuman (1972) for unconfined aquifers, follow the same general procedures.

Hydraulic Test Methods for Aquifers

The aquifer test methods discussed in this section are for the simplest geohydrologic site conditions. Discharge is assumed to be constant; the pumping well is assumed to be a line source. Therefore, wellbore storage is ignored, and the aquifer is assumed to be homogeneous, isotropic, and areally extensive. Solutions to these principal methods are straightforward and type curves are widely available.

Nonleaky Confined Conditions

Solutions to flow conditions induced by discharge from a well in a nonleaky confined or artesian aquifer are considered first. Though based on simple boundary conditions, the solutions to the methods discussed here are useful when applied to appropriate geohydrologic conditions. The methods may also be applied to obtain a preliminary estimate of hydraulic properties as discussed by Bedinger and others (1988) for a test in which geohydrologic conditions are not well known, or for a qualitative examination of aquifer test data to aid in selecting an appropriate method or model.

Theis Nonequilibrium

The solution of Theis (1935) for the change in distribution of head near a well being pumped revolutionized aquifer test methodology and the study of aquifer hydraulics. Although about 50 years old, Theis' method is still the most widely referenced and applied aquifer test method. The Theis solution is the basis and limiting case for solutions to the head distribution in many geohydrologic situations.

Assumptions:

1. The pumping well discharges at a constant rate, Q.

2. The pumping well is of infinitesimal diameter and fully penetrates the aquifer.

3. The nonleaky confined aquifer is homogeneous, isotropic, and areally extensive.

4. The discharge from the pumping well is derived exclusively from storage in the aquifer.

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Implicit in these assumptions are the conditions of radial flow; that is, there are no vertical components of flow and no dewatering of the aquifer. The geometry of the assumed aquifer and well conditions are shown in Figure 4. The Theis (1935) nonequilibrium solution is:

$$s = \frac{Q}{4\pi T} \int_u^\infty \frac{e^{-y}}{y} \, dy \tag{6}$$

and

$$u = \frac{r^2 s}{4Tt} \tag{7}$$

where

$$\int_u^\infty \frac{e^{-y}}{y} \, dy = W(u) = -0.577216 - \log_e u + u$$
$$- \frac{u^2}{2!2} + \frac{u^3}{3!3} - \frac{u^4}{4!4} + \ldots \ldots \tag{8}$$

Application:

The integral expression in equations 6 and 8 cannot be evaluated analytically, but Theis (Wenzel, 1942) devised a graphical procedure to solve for the two unknown parameters, transmissivity, T, and storage coefficient, S, where

$$s = \left(\frac{Q}{4\pi T}\right) W(u) \tag{9}$$

and

$$u = \frac{r^2 s}{(4Tt)} \tag{10}$$

The graphical procedure is based on the functional relations between W(u) and s, and between u and t, or t/r^2.

Steps to perform the Theis procedure are:

1. A type curve illustrating the values of W(u) versus values of 1/u is plotted on logarithmic scale graph paper (Figure 5). This plot is referred to as the type curve plot. Values of W(u) for values of 1/u from 10^{-1} to 9×10^{14} are tabulated by Reed (1980). Values of W(u) for values of 1/u from 10^{-15} to 9.9 are tabulated in Ferris and others (1962), and in Lohman (1972).

2. On logarithmic tracing paper of the same scale and size as the W(u) versus 1/u curve, values of drawdown, s, are plotted on the vertical coordinate versus either time, t, on the horizontal
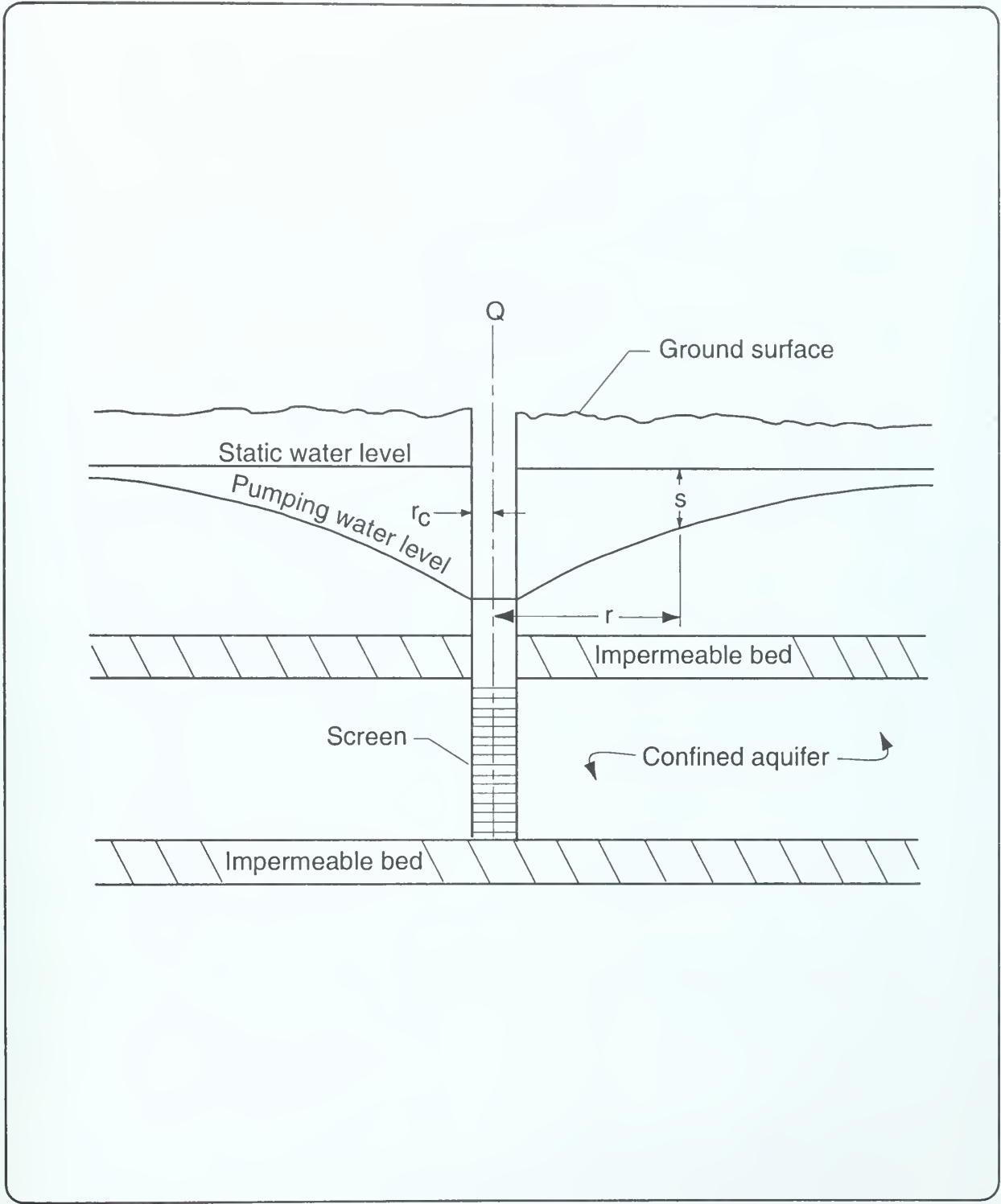
Figure 4. Sections through a pumping well in a nonleaky confined aquifer (After Fetter, 1988).
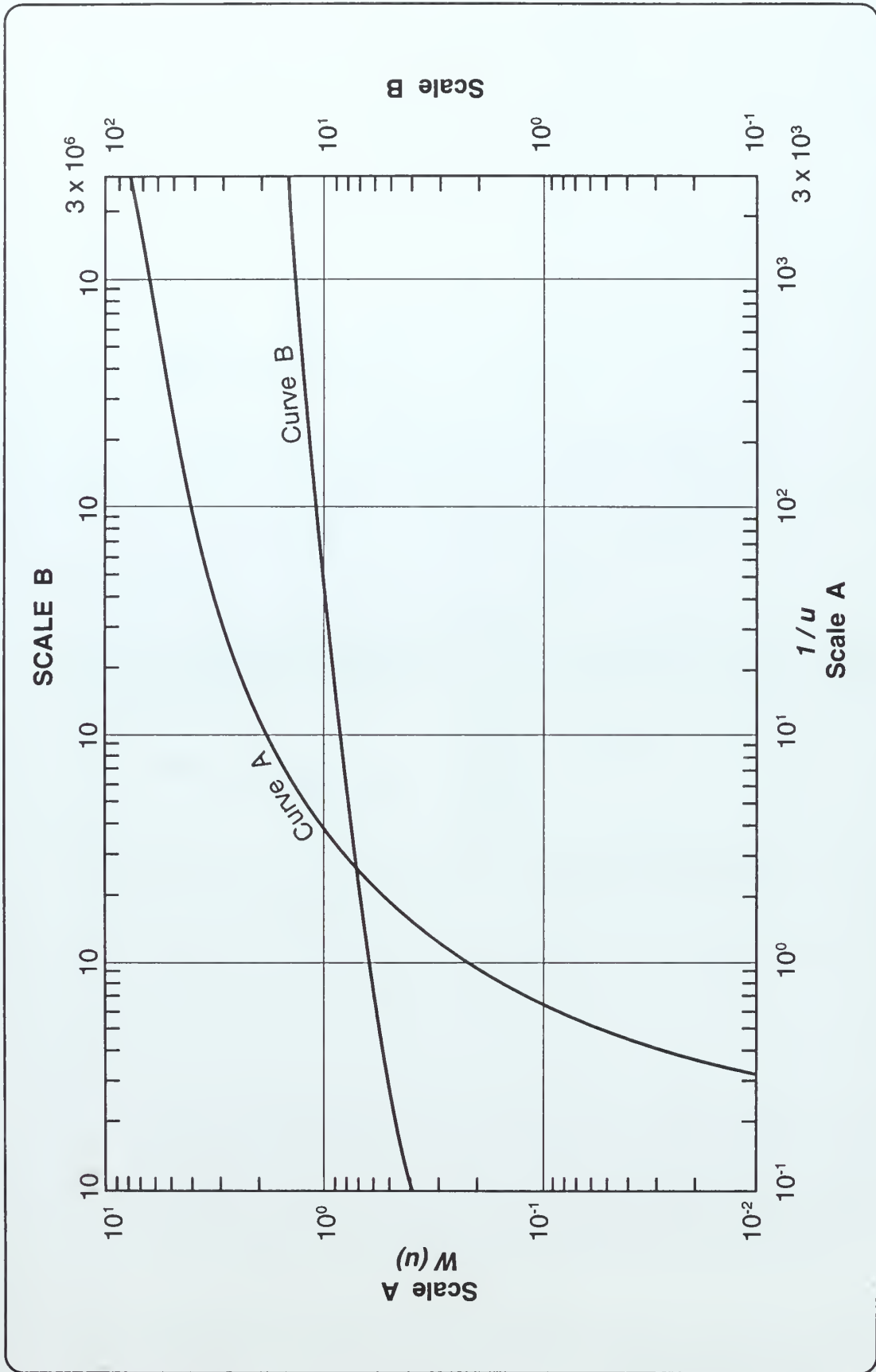
Figure 5. Theis nonequilibrium type curve of dimensionless drawdown, W(u), as a function of dimensionless time, (1/u), for constant discharge from a nonleaky confined aquifer (From Reed, 1980).

coordinate if an observation well is used, or versus t/r^2 on the horizontal coordinate if more than one observation well is used. This plot is referred to as the data plot (Figure 6). Alternatively, the type curve can be plotted as W(u) versus u and the data plot ted as drawdown, s, versus 1/t or r^2/t.

3. The data plot is overlain on the type curve plot and, while the coordinate axes of the two plots are held parallel, the data plot is shifted to a position that represents the best fit of the aquifer test data to the type curve (Figure 6).

4. An arbitrary point, referred to as the match point (Figure 6), is selected anywhere on the overlapping part of the plots and the W(u), 1/u, s, and t coordinates of this point are recorded.

5. Using the coordinates of the point, the transmissivity and storage coefficient are determined from the following equations:

$$T = \frac{QW(u)}{(4\pi s)} \tag{11}$$

and

$$S = \frac{4\,Tut}{r^2} \tag{12}$$

Application of the curve-matching procedure to aquifer test data is discussed by Lohman (1972).

Modified Nonequilibrium

<u>Assumptions:</u>

The straight line method, also called the modified nonequilibrium method, is a solution when u is small and the Theis solution can be approximated by the first two terms on the right side of equation 8.

<u>Solution:</u>

Cooper and Jacob (1946) and Jacob (1950) recognized that in the series of equation 8, the sum of the terms beyond $\log_e u$ is not significant when $u = r^2S/4Tt$ becomes small, $\leq$ about 0.01. The value of u decreases with increasing time, t, and decreases as the radial distance, r, decreases. Therefore, for large values of t and reasonably small values of r, the terms beyond $\log_e u$ in equation 8 may be neglected. The Theis equation can then be written as:

$$s = \frac{Q}{4\pi T}\,[-0.577216 - \log_e \frac{r^2 s}{4Tt}] \tag{13}$$

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Figure labels:

$\log_{10} t$

Data plot

$\log_{10} \dfrac{Q}{4\pi T}$

$\log_{10} s$

Aquifer-test data

Match point
+
coordinates
$W(u)$, $1/u$, s, t

$\log_{10} W(u)$

$\log_{10} \dfrac{4T}{r^2 S}$

Type-curve plot

$\log_{10} \dfrac{1}{u}$
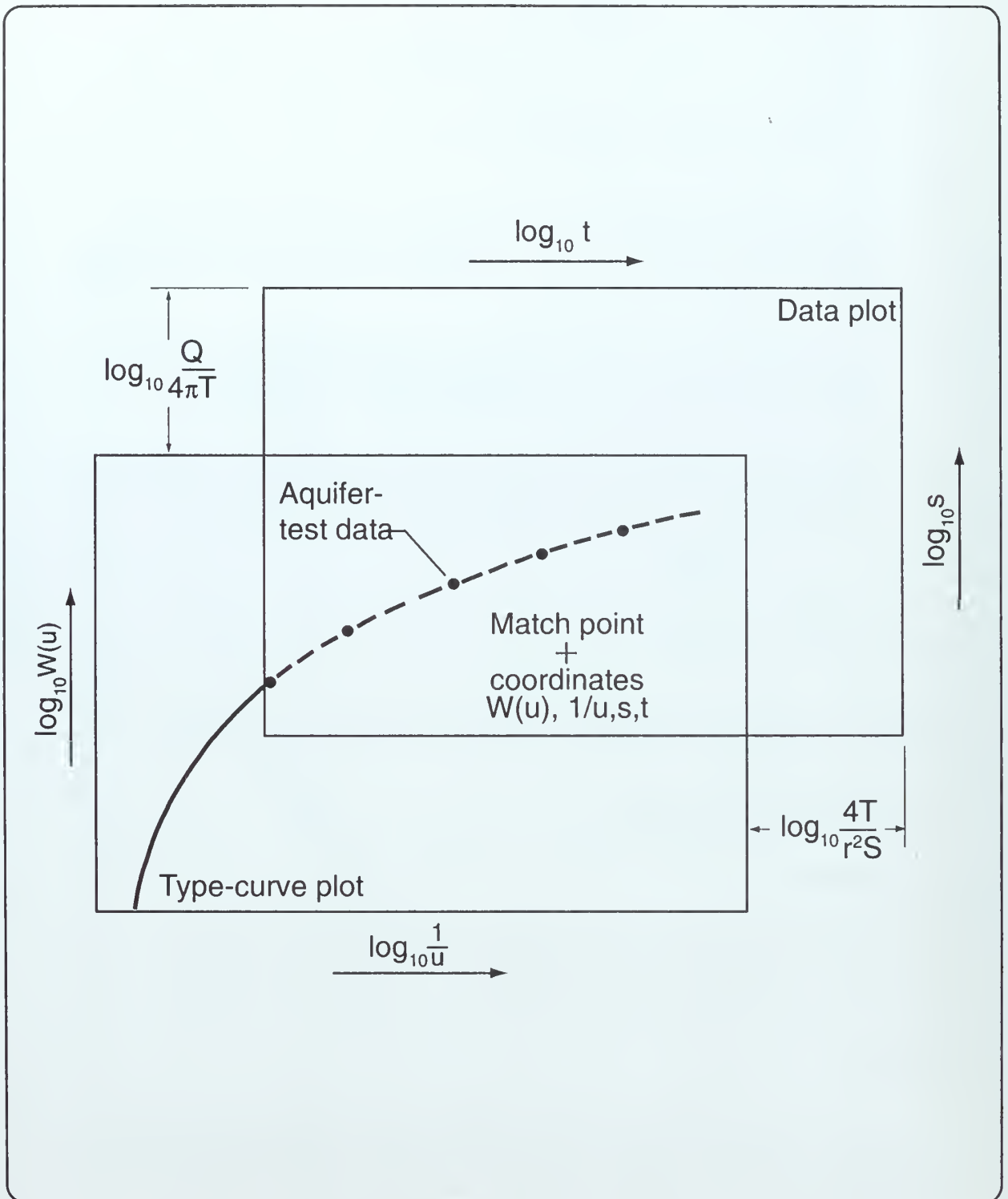
Figure 6. Relation of $W(u)$, $1/u$ type curve and s, t data plot (After Stallman, 1971).

from which Lohman (1972) derives the following equations:

$$T = \frac{2.30 Q}{4 \pi \Delta s / \Delta \log_{10} t} \tag{14}$$

which applies at constant radius and

$$T = \frac{2.30 Q}{2 \pi \Delta s / \Delta \log_{10} r} \tag{15}$$

which applies at constant time. Equation 15 is the same as the Theim (1906) equation.

Application:

Equation 14 can be used to determine transmissivity, T, by plotting drawdown, s, at a specified distance on the arithmetic scale and time, t, on the arithmetic scale versus the distance of the observation wells from the pumping well on the logarithmic scale. By choosing the drawdown, Δs_t or s_r, to be that which occurs over a log cycle,

$$\Delta \log_{10} t = \log_{10} \left(\frac{t_2}{t_1} \right) = 1 \tag{16}$$

and

$$\Delta \log_{10} r = \log_{10} \left(\frac{r^2}{r^1} \right) = 1 \tag{17}$$

equation 14 then becomes

$$T = \frac{2.3 Q}{4 \pi \Delta s_t} \tag{18}$$

and equation 15 then becomes

$$T = - \left(\frac{2.3 Q}{2 \pi \Delta s_r} \right) \tag{19}$$

The coefficient of storage can be determined from these semilog plots of drawdown by a method proposed by Jacob (1950) where

$$s = \frac{2.30 Q}{4 \pi T} \log_{10} \frac{2.25 T t}{r^2 s} \tag{20}$$

taking s = 0 at the zero drawdown intercept of the straight line semilog plot of time or distance versus drawdown

$$S = \frac{2.25 T t}{r^2} \tag{21}$$

where either r or t is the value at the zero drawdown intercept.

Theis Recovery

A useful corollary to the Theis nonequilibrium method was devised by Theis (1935) for the analysis of the recovery of the water level in a control well. The water level in a pumping well that is shut down after being pumped for a known time will recover at a rate that is the inverse of the rate of drawdown. The residual drawdown at any instant will be the same as if the discharge of the well had been continued and a recharge well of the same flow had been introduced at the same point at the instant discharge stopped. The residual drawdown at any time during the recovery period is the difference between the measured water level and the pumping water level interpreted from the measured trend prior to the stoppage of pumping as discussed by Bedinger and others (1988). These relationships are shown in Figure 7. The residual drawdown, s', at any instant can be expressed as:

$$s' = \frac{Q}{4\pi T} \left[\int_u^\infty \frac{e^{-u}}{u}\,du \;-\; \int_{u'}^\infty \frac{e^{-u'}}{u'}\,du' \right] \tag{22}$$

The Theis recovery method is applied in much the same way as the drawdown method. From the time since pumping ceased t' becomes large, the semilog plot of residual drawdown s', and the ratio of time since pumping started to time since pumping ceased t/t', is plotted on the logarithmic scale. Transmissivity can be calculated from the following equation:

$$T = \frac{2.3\,Q}{4\pi\Delta s'} \log_{10}\left(\frac{t}{t'}\right) \tag{23}$$

Slug Test

The method for estimating transmissivity by injecting a given quantity or slug of water into a well was originally described by Ferris and Knowles (1954). Included in this category of tests are methods for determining transmissivity in a slug well by determining the response to an instantaneous change in head in the well. Head change may be induced by injection of water, bailing of water from the well, rapid removal of a solid cylinder from beneath the water level in a well to which the water level was in equilibrium, or application of pressure to the volume of water stored in a shut in well.

Figure 7. Graph showing drawdown, recovery, and residual drawdown (After Dawson and Istok, 1991).

Figure 8. Section through a pumping well in which a slug of water is suddenly injected (After Reed, 1980).

Assumptions:

1. A volume of water is injected into or is discharged from the slug well instantaneously at t=0.

2. The slug well is of finite diameter and fully penetrates the aquifer.

3. Flow is radial in the areally extensive, homogeneous, and isotropic, nonleaky confined aquifer.

The geometry of the slug well and aquifer is shown in Figure 8.

Solution:

The solution presented by Cooper and others (1967) for wells that are not affected by inertially induced, oscillatory water level fluctuations, is for a slug well of finite diameter; application of the solution is by matching of aquifer test data to type curves. The solution and its application have been elaborated on by Bredehoeft and Papadopulos (1980), and Neuzil (1982). The solution of Cooper and others (1967) is

$$
h = \left(\frac{2H_0}{\pi}\right) \int_0^\infty \left\{\left\{ Exp\left(\frac{-\beta u^2}{\alpha}\right) \left\{ J_0\left(\frac{ur}{r_w}\right)\right. \right.\right.
$$
$$
\left[u\gamma_0(u) - 2\alpha\gamma_1(u)\right] - \gamma_0\left(\frac{ur}{r_w}\right)
$$
$$
\left.\left.\left.\left[uJ_0(u) - 2\alpha J_1(u)\right]\right\}/\Delta(u)\right\}\right\} du \qquad (24)
$$

where

$$
\alpha = \frac{r_w^2 S}{r_c^2} \qquad (25)
$$

$$
\beta = \frac{Tt}{r_c^2} \qquad (26)
$$

and

$$
\Delta(u) = \left[uJ_0(u) - 2\alpha J_1(u)\right]^2
$$
$$
+ \left[u\gamma_0(u) - 2\alpha\gamma_1(u)\right]^2 \qquad (27)
$$

The head, H, inside the slug well, obtained by substituting r=rw in equation 24 is

$$
\frac{H}{H_0} = F(\beta, \alpha) \qquad (28)
$$
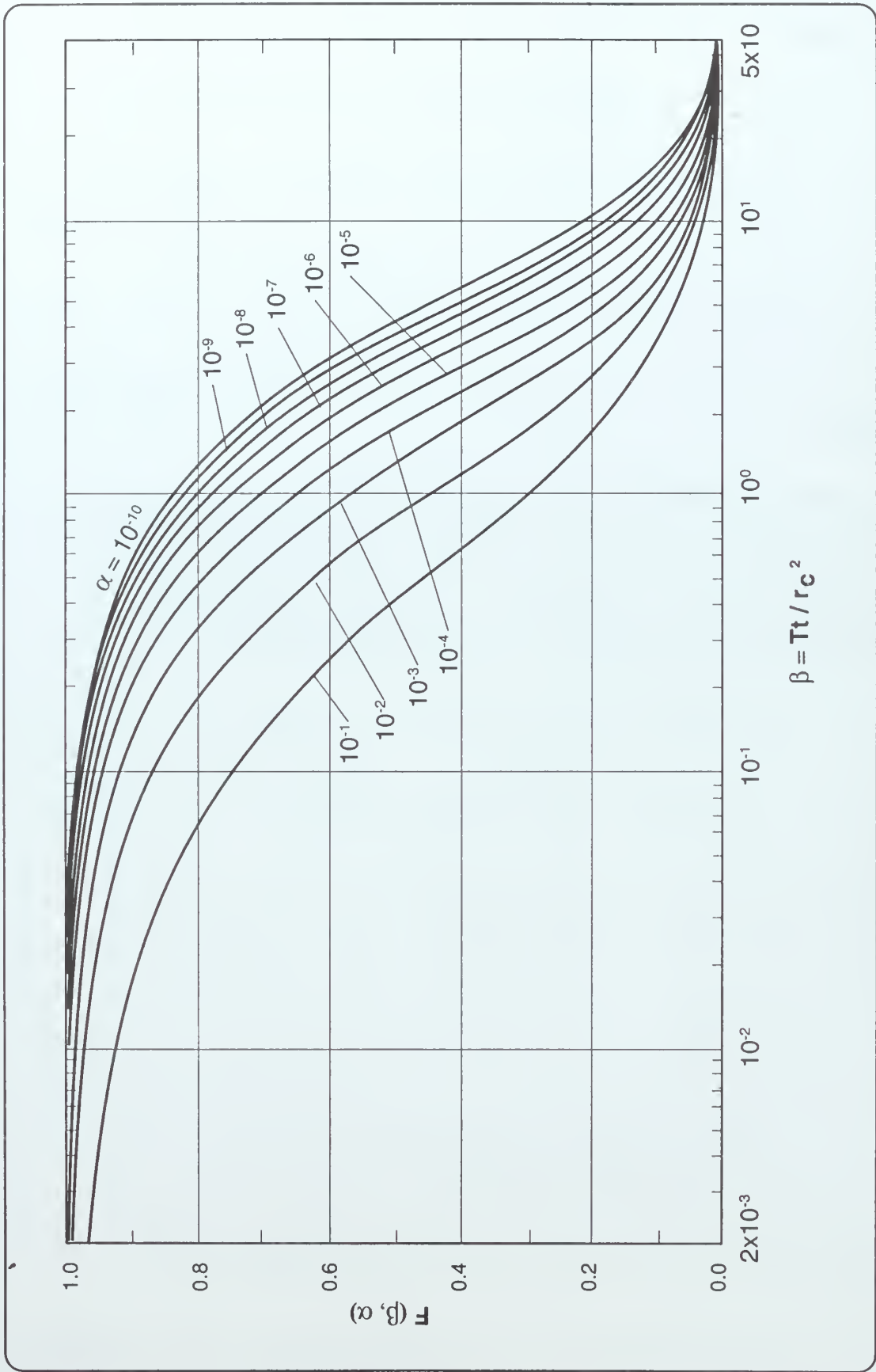
Figure 9. Graph showing ten selected type curves of F (β, α) as a function of β (From Reed, 1980).

The graph plots F (β, α) on the vertical axis (ranging from 0.0 to 1.0) against $\beta = Tt/r_c^2$ on the horizontal axis (ranging from 2×10^{-3} to 5×10). The curves are labeled $\alpha = 10^{-10}$, 10^{-9}, 10^{-8}, 10^{-7}, 10^{-6}, 10^{-5}, 10^{-4}, 10^{-3}, 10^{-2}, 10^{-1}.

where

$$F(\beta, \alpha) = (\frac{8\alpha}{\pi^2}) \int_0^\infty [Exp(\frac{-\beta u^2}{\alpha}) / u\Delta(u)] \, du \qquad (29)$$

The curves generated from equation 28 are plotted in Figure 9.

Application:

The water level data in the slug well, expressed as a fraction of H_0 (that is, H/H_0) are plotted versus time, t, on semilogarithmic graph paper of the same scale as that of the type curve plot. This data plot is overlain on Figure 9 and, while keeping the baselines the same, the data plot is shifted horizontally until a match or interpolated fit of the aquifer test data to a type curve is made. A match point for ß, t, and a is picked on the overlapping part of the plots and the coordinates of this point are recorded. The transmissivity is calculated from

$$T = \frac{\beta r_c^2}{t} \qquad (30)$$

and the storage coefficient from

$$S = \frac{\alpha r_c^2}{r_w^2} \qquad (31)$$

As pointed out by Cooper and others (1967), the determination of S by this method has questionable reliability because of the similar shape of the curve, whereas the determination of T is not as sensitive to choosing the correct curve. Figure 9 is plotted from data from two sources (Cooper and others, 1967; and Papadopulos and others, 1973). Tables of the F(ß,α) are given in Cooper and others (1967) for values of ß from 10^{-3} to 2.15×10^2 and for values of α from 10^{-6} to 10^{-10} in order to apply the method to formations having a very small storage coefficient.

Although the method applies to radial flow in a nonleaky confined aquifer, the method has been applied to partially penetrating wells where the screened interval is much larger than the well radius. In a stratified aquifer where the vertical permeability is much smaller than the horizontal permeability, the flow for a test of short duration can be assumed to be virtually radial. The transmissivity thus derived would apply to the part of the aquifer in which the well is screened or open.

Bredehoeft and Papadopulos (1980) adapted the method for application to formations of very low permeability and extended the range of F(ß,α). Bredehoeft and Papadopulos (1980) described a technique of pressurizing a shut in well in low permeability rocks that decreases the response time by orders of magnitude. Neuzil (1982) determined that the slug test method for very low permeability formations by Bredehoeft and Papadopulos (1980) does not assure the condition of approximate equilibrium necessary at the start of the slug test; Neuzil (1982) also determined that the compliance of the shut in well and associated piping, which determines the response time, can be substantially larger than the compressibility of water alone. Neuzil (1982) presented a modified procedure and testing arrangement for slug tests in low permeability formations. The region adjacent to the wellbore may be altered by the addition of drilling mud, precipitation of scale, well stimulation, or gravel pack. Moench and Hsieh (1985) examined the altered region or skin adjacent to the wellbore and concluded that pressure tests may be markedly affected by the altered skin. Moench and Hsieh (1985) determined that standard methods of analysis are adequate for open well slug tests.

Airlift Test

The data analysis procedure is outlined by Kruseman and Ridder (1991). This method is very similar to the Cooper and Jacob method (1946), but was developed by Aron and Scott (1965) for a well in a confined aquifer, with the exception that it is assumed the discharge rate decreases with time with the sharpest decrease occurring soon after the start of pumping.

The procedure involves injecting pressurized air down the well to lift water to the surface as shown in Figure 10, while recording drawdown and discharge over time. These data are plotted on semi-log paper with drawdown divided by production rate (s/Q) plotted on the vertical scale and log time plotted on the horizontal scale. This usually results in a straight line; the slope of the line is then used to calculate the transmissivity as shown in Figure 11. A more detailed description of the equipment setup and layout is presented by Driscoll(1986).

The airlift test procedure has been applied on numerous projects as noted by Doubek and Beale (1992). Some of the advantages of this method are that the test can be conducted with standard exploration drilling equipment, water level measurements and transmissivities can be obtained, and the test is less costly than some other methods. Some disadvantages of this method are that the test cannot be used to obtain storativities, stresses only affect zones close to the pumping well, and the analytical method must meet discharge rate and other constraints as noted by Cooper and Jacob (1946) for applicability.
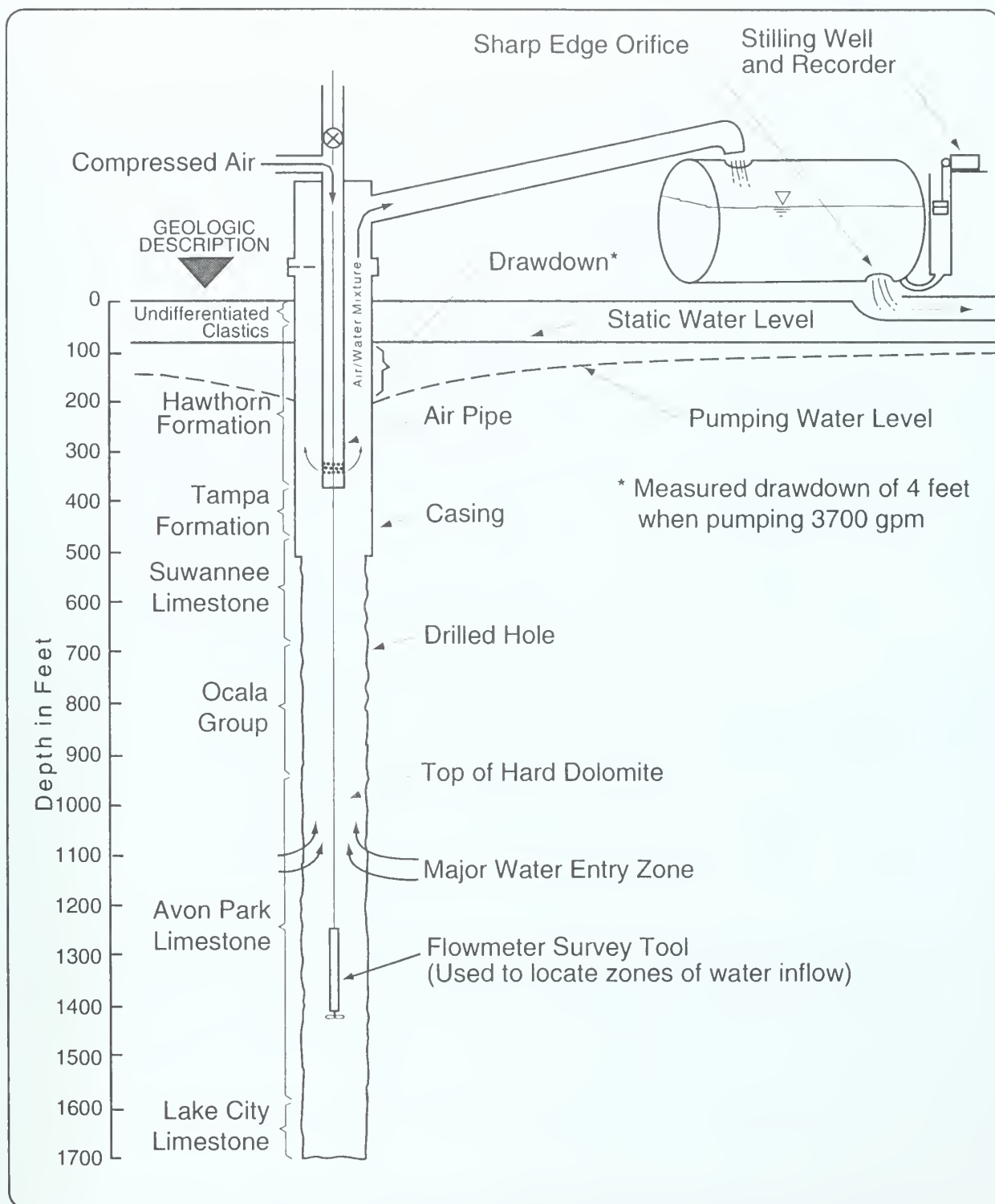
Figure 10. Schematic representation of airlift pumping test (Baski, 1978).
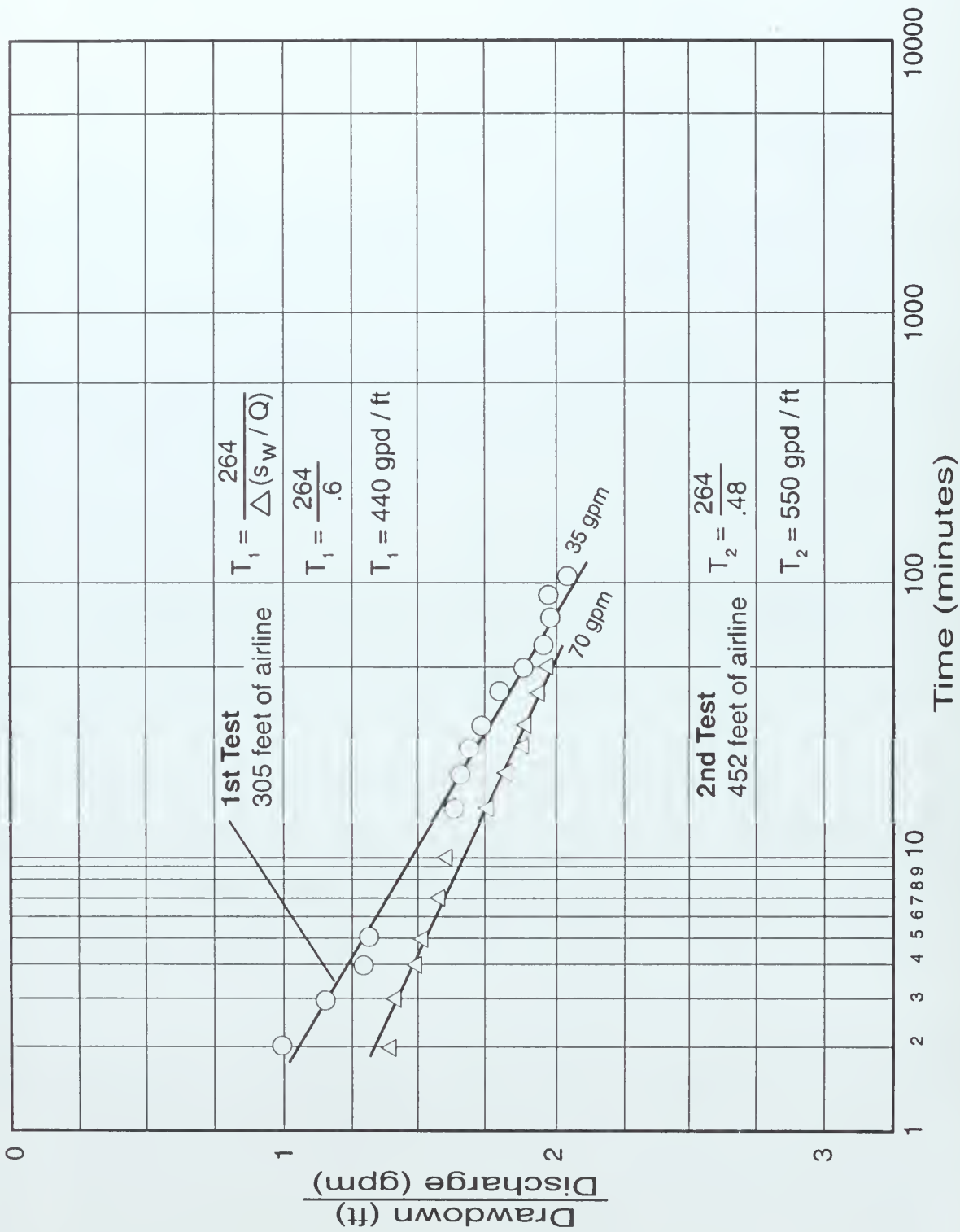
Figure 11. Airlift pump test drawdown plots (Baski, 1978)

The plot contains the following annotations:

1st Test
305 feet of airline

$$T_1 = \frac{264}{\Delta(s_w/Q)}$$

$$T_1 = \frac{264}{.6}$$

$$T_1 = 440 \text{ gpd/ft}$$

35 gpm

70 gpm

2nd Test
452 feet of airline

$$T_2 = \frac{264}{.48}$$

$$T_2 = 550 \text{ gpd/ft}$$

Axis labels:
- X-axis: Time (minutes)
- Y-axis: $\frac{\text{Drawdown (ft)}}{\text{Discharge (gpm)}}$

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Leaky Confined Conditions

Confining beds above or below the aquifer commonly provide water to the aquifer by leakage when the aquifer is pumped. Methods that account for leakage will be discussed next.

Leaky Confining Bed Without Storage

Assumptions:

1. Pumping well discharge is at a constant rate, Q.

2. Pumping well is of infinitesimal diameter and fully penetrates the confined aquifer.

3. Confined aquifer is overlain or underlain everywhere by a leaky confining bed having uniform hydraulic conductivity, K' and thickness, b'.

4. Leaky confining bed is overlain or underlain everywhere by an infinite source bed with a constant hydraulic head.

5. Hydraulic gradient across the leaky confining bed changes instantaneously with a change in head in the confined aquifer (no release of water from storage in the leaky confining bed).

6. Flow in the confined aquifer is two dimensional and radial in the horizontal plane; flow in the leaky confining bed is vertical.

The nonequilibrium technique of Hantush and Jacob (1955), though a simplification of a leaky flow system, is widely applied as discussed by Bedinger and others (1988). The method assumes an unlimited supply of water from the overlying or underlying beds, but no release of water from storage in the confining beds.

The geometry of the assumed well and aquifer system is shown is Figure 12.

The assumption of no release of water from storage in the leaky confining bed may usually be met at early times before water is yielded from the confining bed and at late times when the system is near steady state. The assumption may also estimate conditions for thin confining beds.
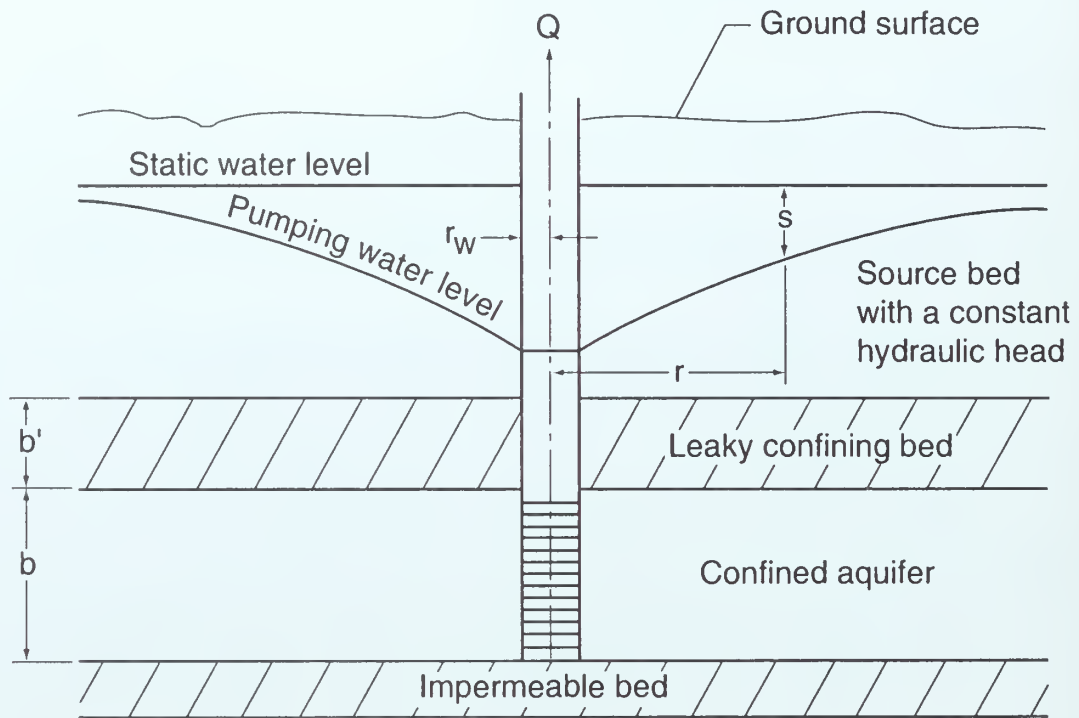
Figure 12. Section through a pumping well in a leaky confined aquifer without storage of water in the leaky confining bed (After Reed, 1980).

<u>Solution:</u>

The solution for the conditions stated as given by Hantush and Jacob (1955) are:

$$s = \frac{Q}{4\pi T} \int_u^\infty \frac{e^{-z} - (r^2/4B^2z)}{z} \, dz$$
$$= \frac{Q}{4\pi T} \, W(u, r/B) \tag{32}$$

where

$$u = \frac{r^2 S}{4Tt} \tag{33}$$

and

$$B = \sqrt{\left(\frac{Tb'}{K'}\right)} \tag{34}$$

Cooper (1963) expressed the solution as:

$$s = \frac{Q}{4\pi T} \int_u^\infty \frac{e^{-y} - (v^2/y)}{y} \, dy$$
$$= \frac{Q}{4\pi T} \, L(u, v) \tag{35}$$

with

$$v = \frac{r}{2} \sqrt{\left(\frac{K'}{Tb'}\right)} \tag{36}$$

The notations of Hantush and Jacob (1955) and Cooper (1963) are included here because type curves, tables, and data analyses using both are found in the literature. Hantush and Jacob (1955) point out that as B approaches infinity, that is as leakage decreases, equation 32 approaches the Theis equation (Equation 6). The L(u,v) of Cooper (1963) is called the leakance function of u and v.

Hantush and Jacob (1954) noted that flow in a leaky confined aquifer is three dimensional, but if the hydraulic conductivity of the aquifer is sufficiently greater than that of the leaky confining bed, the flow may be assumed to be vertical in the confining bed and radial in the aquifer. This relation has been quantified by Hantush (1967a) for the condition b/B < 0.1. Assumption 5, that there is no change in storage of water in
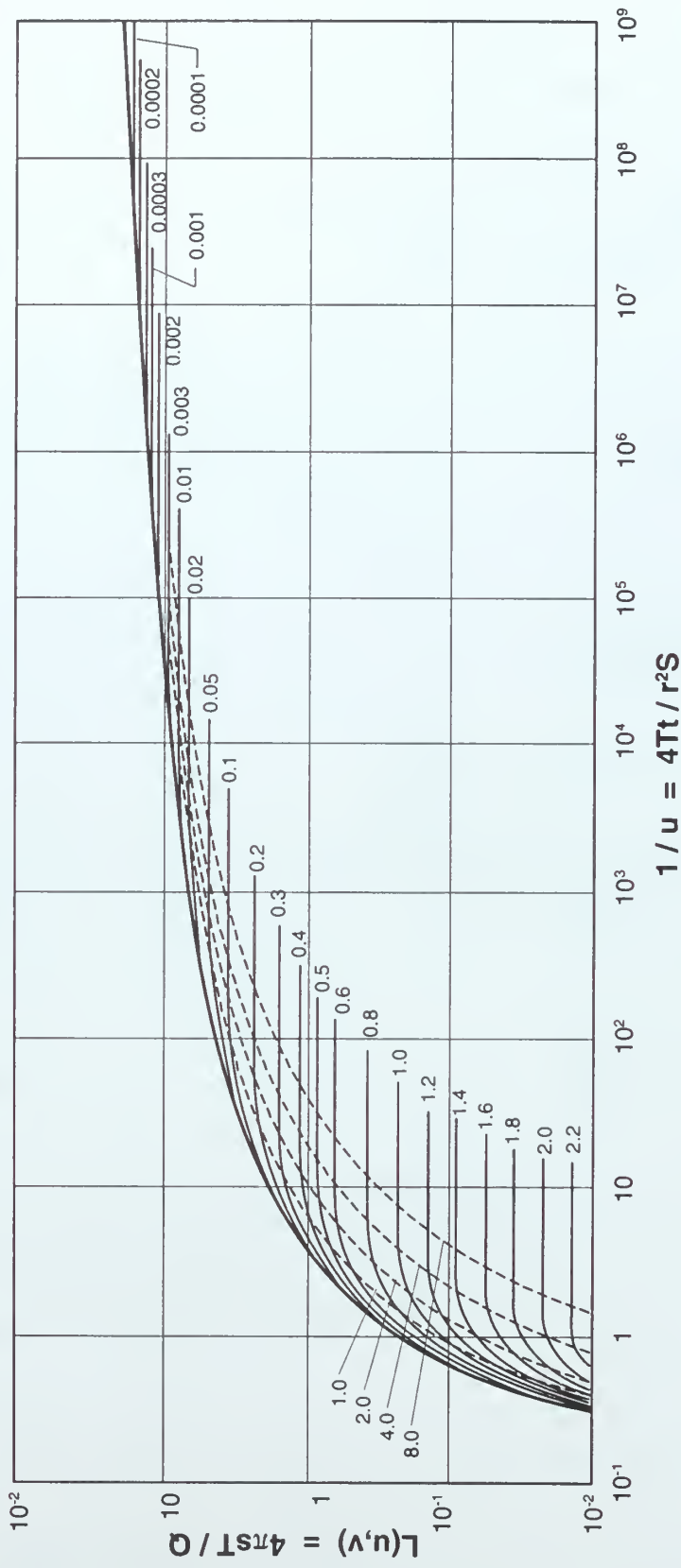
Figure 13. Type curve of W(u, r / B) - L(u,v) as a function of 1 /u (From Loman, 1972).

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the confining bed, was investigated by Neuman and Witherspoon (1969a). They concluded that this assumption would not affect the solution if ß < 0.01, where

$$\beta = \frac{r}{4b} \sqrt{\left(\frac{K'S'_s}{KS_s}\right)} \tag{37}$$

Assumption 4, that there is no drawdown in water level in the source bed, was also examined by Neuman and Witherspoon (1969a). They indicated that drawdown in the source bed is justified when $T_S > 100T$, where T_S represents the transmissivity of the source bed and would have negligible effect on the drawdown in the pumped aquifer for short times; that is, when $Tt/r^2S < 1.6$ ß^2/(r/B)^4.

Figure 13 shows plots of dimensionless drawdown compared to dimensionless time from Reed (1980) using the notations of Hantush, Jacob, and Cooper (1963).

Application:

Aquifer test data may be plotted in two ways. For the first method, measured drawdown in any one well is plotted versus t/r^2; the data are matched to the solid line type curves in Figure 13. The data points are aligned with the solid-line type curves either on one of them or between them. Using the notation of Hantush and Jacob (1955), the parameters are then computed from the coordinates of the match points $(t/r^2, s)$ and $[1/u, W(u, r/B)]$, and an interpolated value of r/B from the equations

$$T = \frac{Q}{4\pi} \; \frac{W(u, r/B)}{s} \tag{38}$$

$$S = 4T \left(\frac{t/r^2}{1/u}\right) \tag{39}$$

and

$$\frac{K'}{b'} = \frac{T}{B^2} \tag{40}$$

Using the notation of Cooper (1963), the parameters are computed from the coordinates of the match points $(t/r^2, s)$ and $[1/u, L(u, v)]$, and an interpolated value of v from equations

$$T = \left(\frac{Q}{4\pi}\right) \left(\frac{L(u, v)}{s}\right) \tag{41}$$
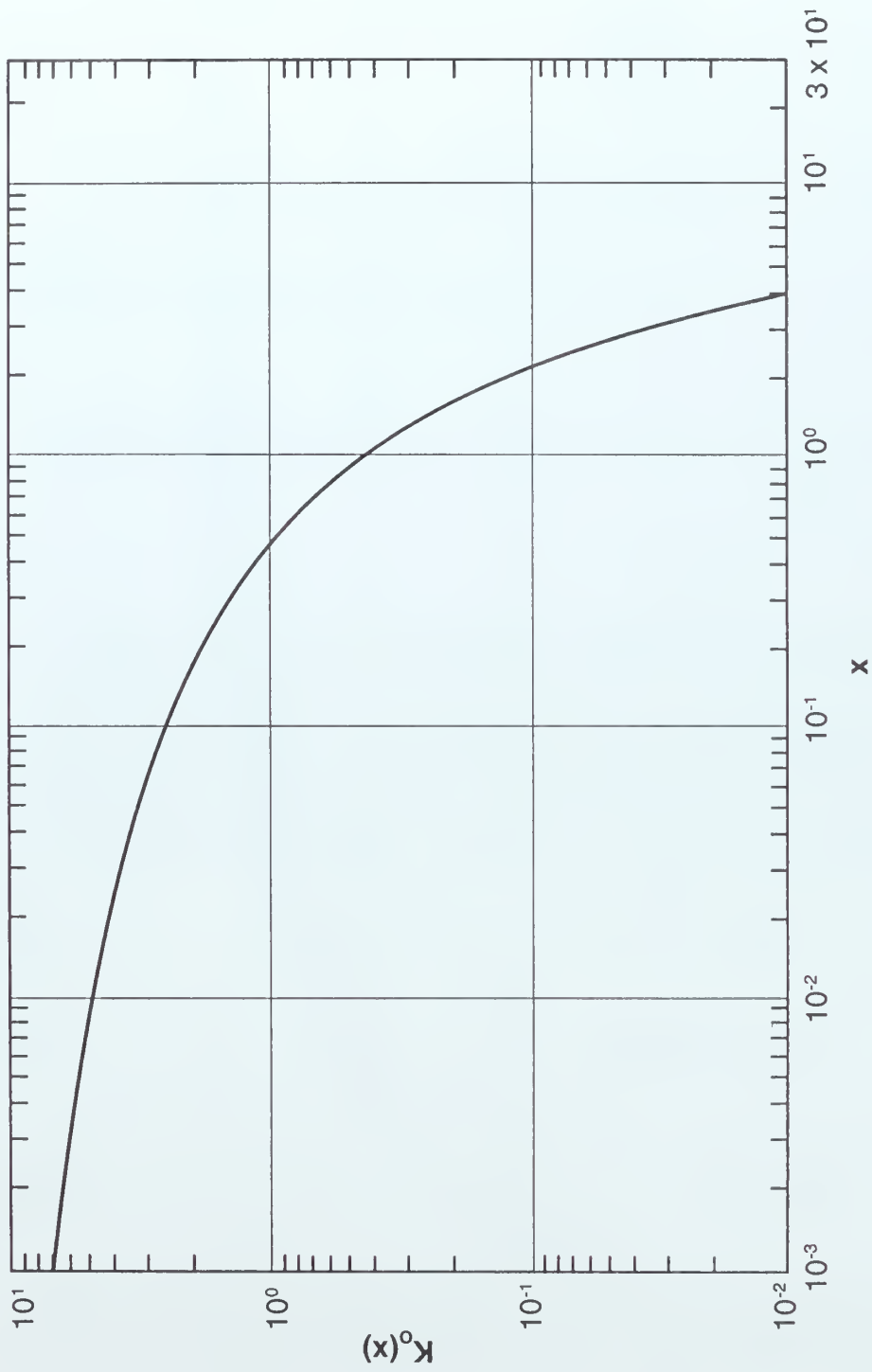
Figure 14. Type curve of the Bessel function $K_0'x(x)$ as a function of x (After Reed, 1980).

$$S = 4T \left(\frac{t/r^2}{1/u} \right) \tag{42}$$

and

$$\frac{K'}{b'} = 4T \left(\frac{v^2}{r^2} \right) \tag{43}$$

This method was used by Cooper (1963) and the data and analysis of Cooper is cited by Lohman (1972).

Cooper (1963) devised a second method as discussed by Bedinger and others (1988) by which drawdown measured at the same time but in different wells at different distances can be plotted versus t/r^2 and matched to the dashed curves of Figure 13. The data are matched so as to align with the dashed line curves, either on one or between two of them. From the match point coordinates $(s,t/r^2)$ and $[W(u,r/B), 1/u]$ and an interpolated value of v^2/u, T and S are computed from equations 41 and 42 and the remaining parameter from

$$\frac{K'}{b'} = S \left(\frac{v^2/u}{t} \right) \tag{44}$$

Equilibrium Method relates to the fact that the zone $v^2/u \geq 8$ and $W(u,r/B) \geq 0.02$ in the method of Hantush (1956) corresponds to steady state conditions. The drawdown in the steady state zone is given by Jacob (1946):

$$s = \left(\frac{Q}{2\pi T} \right) K_0 (x) \tag{45}$$

where $K_0(x)$ is the zero order modified Bessel function of the second kind and

$$x = r \sqrt{\left(\frac{K'}{Tb'} \right)} \tag{46}$$

Data for steady state conditions can be analyzed using the type curve in Figure 14. The drawdowns are plotted versus r and matched to the type curve. After choosing a convenient match point with coordinates (s,r) and $[K_0(x),x]$, the parameters are computed from the equations:

$$T = \left(\frac{Q}{2\pi s} \right) K_0 (x) \tag{47}$$

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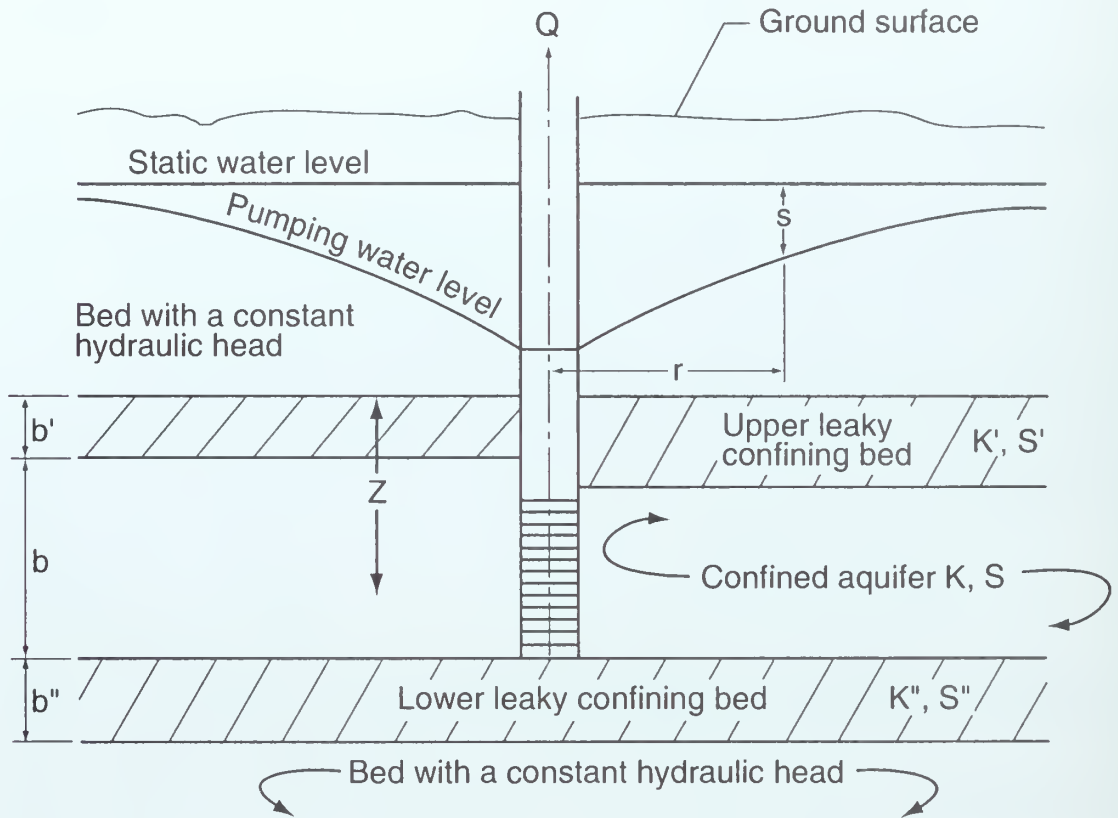
Figure 15. Section through a pumping well in a leaky confined aquifer with storage of water in confining beds (After Dawson and Istok, 1991).

and

$$\frac{K'}{b'} = \frac{x^2 T}{r^2} \qquad (48)$$

Leaky Confining Bed With Storage

Assumptions:

1. Pumping well discharge is at a constant rate, Q.

2. Pumping well is of infinitesimal diameter and fully penetrates the confined aquifer.

3. Confined aquifer is overlain and underlain everywhere by leaky confining beds having uniform values of hydraulic conductivity, K' and K", thickness, b' and b", and storage coefficient, S' and S".

4. Leaky confining beds are overlain and underlain everywhere by infinite beds with constant hydraulic heads.

5. Flow in the confined aquifer is two dimensional and radial in the horizontal plane; flow in the leaky confining beds is vertical.

Hantush (1960) presented solutions for determining head in response to discharge from leaky confined aquifers where release of water from storage in the confining beds is taken into account. Release of water from storage in confining beds may be substantial in a number or geohydrologic situations, such as where the confining beds are thick or where the upper confining bed contains a water table. Also, release of water from storage may be substantial for short durations (t less than both b' S'/10K' and b" S"/10K") in many geohydrologic situations. Release of water from storage in confining beds commonly becomes less significant with time as steady state flow conditions are approached. A complete discussion of the Hantush (1960) methods for the geometry in Figure 15 and for other types of geometry is presented by Reed (1980). The solution of Hantush (1960) for short durations (t less than both b' S'/10K' and b" S"/10K") is:

$$s = (\frac{Q}{4 \pi T}) \, H(u, \beta) \qquad (49)$$
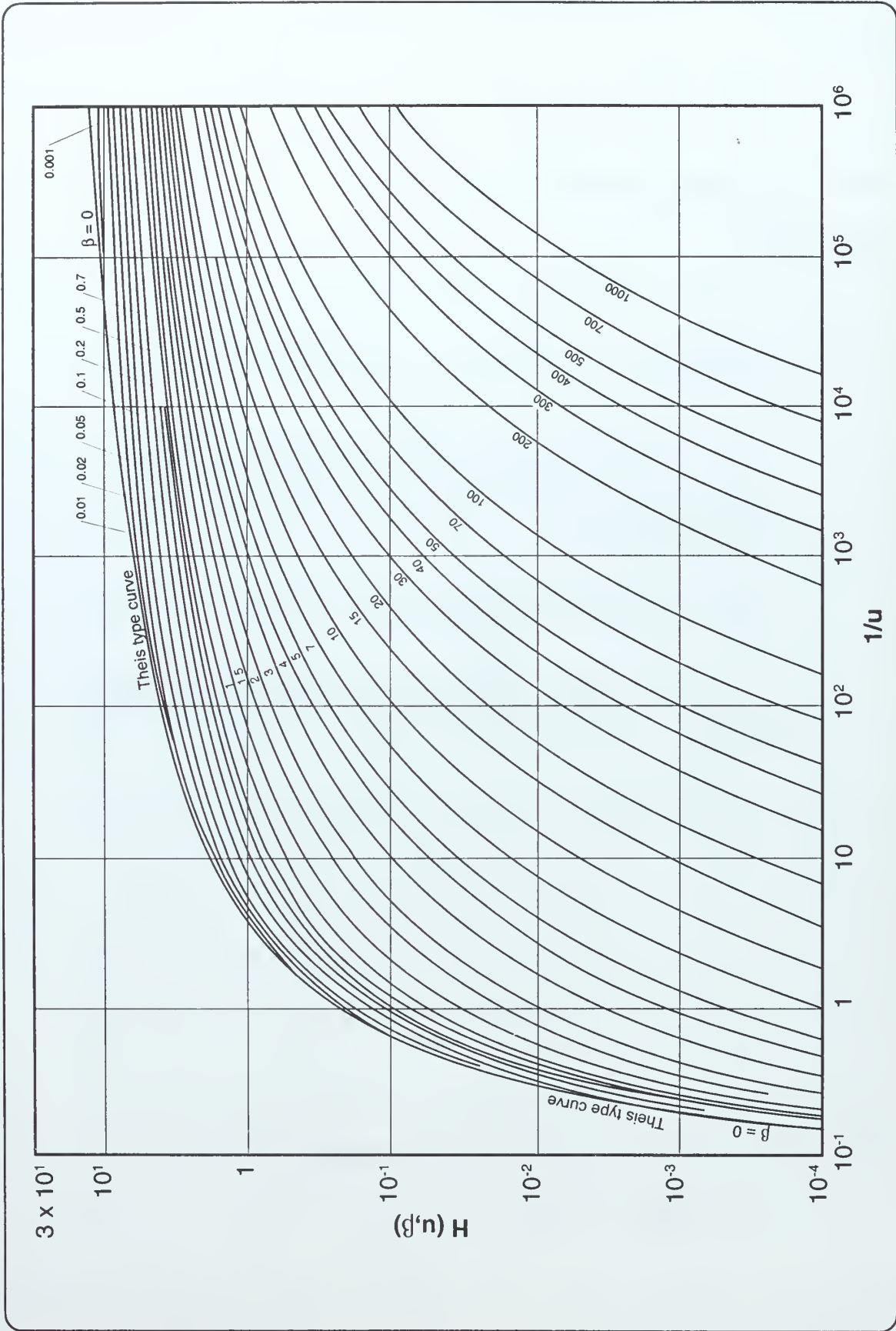
Figure 16. Dimensionless drawdown, H(u, b), as a function of dimensionless time, 1/u, for a well fully penetrating a leaky confined aquifer with storage (From Loman, 1972).

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where

$$u = \frac{r^2 S}{4Tt} \tag{50}$$

and

$$\beta = \frac{r}{4} \left(\sqrt{\left(\frac{K'S'}{b'TS} \right)} + \sqrt{\left(\frac{K''S''}{b''TS} \right)} \right) \tag{51}$$

and

$$H(u, \beta) = \int_u^\infty \frac{e^{-y}}{y} \; erfc \; \frac{\beta\sqrt{u}}{\sqrt{y(y-u)}} \; dy \tag{52}$$

and

$$erfc(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-y^2} \; dy \tag{53}$$

Lohman (1972) points out that the versatility of equations 49 through 53 is because they are the general solution for determining the drawdown distribution in all confined aquifers as discussed by Bedinger and others (1988), whether they are leaky or nonleaky. That is, ß approaches zero as K' and K'' approach zero, and equation 48 becomes the Theis equation 9.

Application:

The method can be applied by plotting drawdown versus t/r^w and superposing the data plot on the type curve plot of H(u,ß) versus 1/u as shown in Figure 16. An example of the application of this method using data from an aquifer test is presented by Lohman (1972).

Unconfined Conditions

Conditions governing drawdown due to discharge from an unconfined aquifer differ markedly from those due to pumping from a nonleaky confined aquifer. Difficulties in deriving analytical solutions to the hydraulic head distribution in an unconfined aquifer result from the following characteristics:
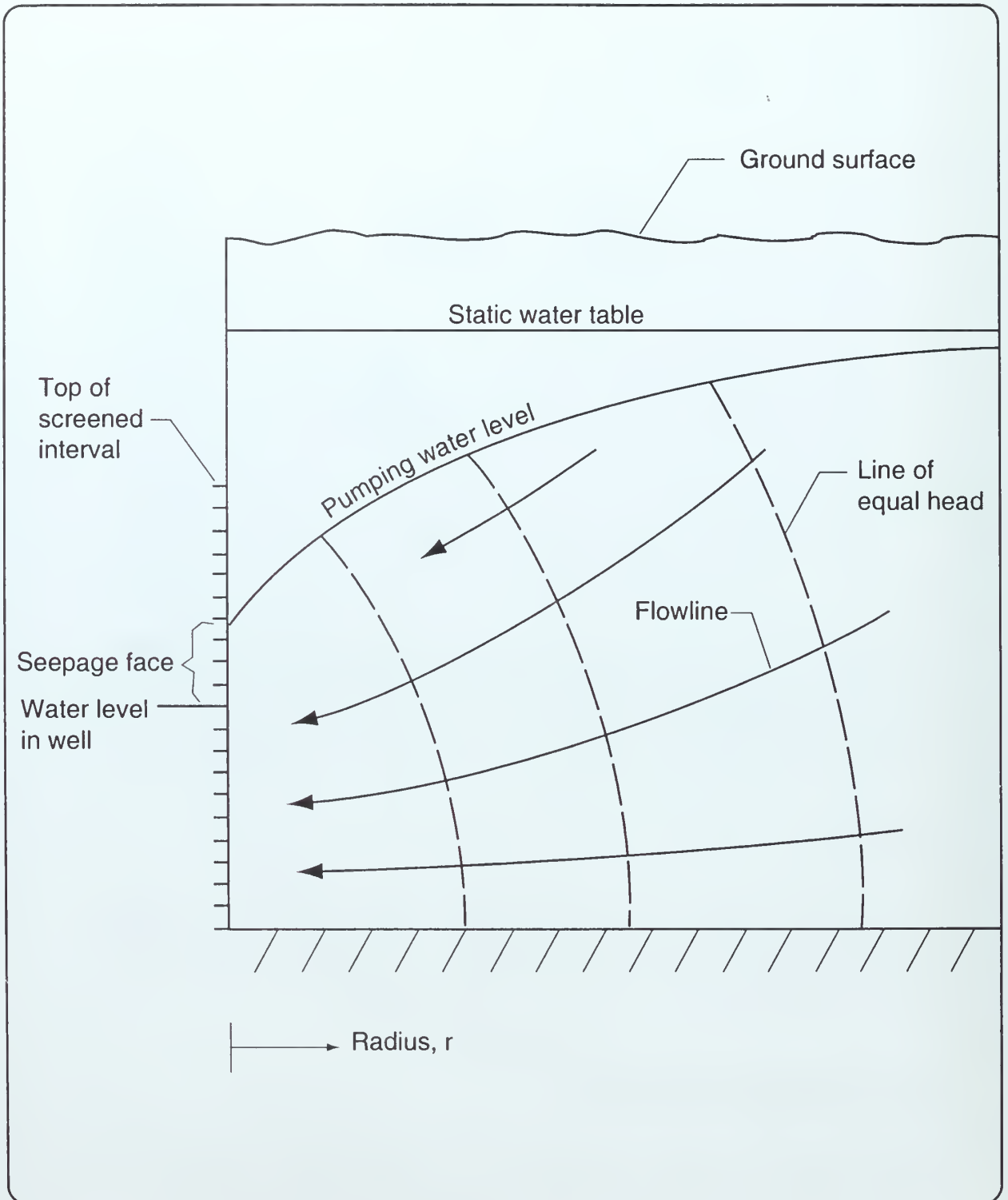
Figure 17. Diagrammatic section through a pumping well in an unconfined
aquifer (After Fetter, 1988).

1. Transmissivity varies in space and time as the water table is drawn down and the aquifer is dewatered.

2. Water is derived from storage in an unconfined aquifer mainly at the free water surface and, to a smaller degree, from each discrete point within the aquifer.

3. Vertical components of flow exist in the aquifer in response to withdrawal of water from a well. These components may be large and are greater near the pumping well and at early times. The diagrammatic section in Figure 17 is through a pumping well in an unconfined aquifer and shows conditions when the pumping well is pumped. If the water level in the pumping well is below the top of the screened interval, a seepage face will be present.

The drawdown curve in an unconfined aquifer in response to an active pumping well follows a typical S-shaped curve. During early times of pumping activity, water level decline is rapid, and water is derived internally from the aquifer by expansion of the water and compaction of the aquifer; head response is similar to that of a confined aquifer. As pumping continues, head response lags that of a confined aquifer. This lag was attributed to slow drainage from the unsaturated zone by many early investigators. However, Cooley and Case (1973) concluded that the unsaturated zone has little effect on flow in the aquifer. Neuman (1972) attributed the lag to delayed response related to vertical components of flow in the aquifer as a function of the radial distance from the pumping well and of time. At later times, the drawdown once again appears to follow the Theis curve.

Solution of Boulton and Neuman

Boulton (1954b and 1963) introduced a mathematical solution to the head distribution in response to pumping an unconfined aquifer. Boulton's solution derives the typical S-shaped curves of unconfined aquifers, but invokes the use of a semiempirical delay index that was not defined on a physical basis as discussed by Bedinger and others (1988). Neuman (1972 and 1975) presented a solution for unconfined aquifers based on well-defined physical properties of the aquifer. Neuman (1975) examined the physical basis for Boulton's delay index ($1/\alpha$) and determined that, for fully penetrating pumping wells, Boulton's solution yielded values of transmissivity, specific yield, and storage coefficient identical to those determined by Neuman. Neuman's method for unconfined aquifers is discussed here. For further information on Boulton's method, the reader is referred to Boulton (1954a and 1963). Application

of Boulton's method to aquifer test data from unconfined aquifers is presented by Prickett (1965) and Lohman (1972).

Assumptions:

1. Pumping well of infinitesimal diameter discharges at a constant rate, Q.

2. Pumping well and observation well are open throughout the thickness of the unconfined aquifer.

3. Unconfined aquifer is areally extensive, homogeneous, and isotropic with vertical hydraulic conductivity, K_z, and horizontal hydraulic conductivity, K_r.

Solution:

The solution of Neuman (1973) for the condition in which the pumping well and the observation well are perforated throughout the saturated section of the aquifer is given by:

$$s(r, t) = \frac{Q}{4\pi T} \int_0^\infty 4y J_0 \ (y\beta^{1/2})$$

$$[u_0(y) + \sum_{n-1}^\infty u_n(y)] \ dy \tag{54}$$

where

$$u_o(y) = \frac{\{1 - Exp \ [-t_s\beta \ (y^2 - \gamma_0^2)]\} \ \tanh \ (\gamma_0)}{\{y^2 + (1 + \sigma) \ \gamma_0^2 - [(y^2 - \gamma_0^2)^2/\sigma]\} \ \gamma_0} \tag{55}$$

and

$$u_n(y) = \frac{\{1 - Exp \ [-t_s\beta \ (y^2 - \gamma_n^2)]\} \ \tan \ (\gamma_n)}{\{y^2 - (1 + \sigma) \ \gamma_n^2 - [(y^2 - \gamma_n^2)^2/\sigma]\} \ \gamma_n} \tag{56}$$

and the terms γ_0 and γ_n are the roots of the equations

$$\sigma\gamma_0 \ \sinh \ (\gamma_0) - (y^2 - \gamma_0^2) \ \cosh \ (\gamma_0) = 0$$
$$where \ \gamma_0^2 < y^2$$
$$shows \ relationship \ of \ quantities \tag{57}$$

and

$$\sigma\gamma_n \sin(\gamma_n) + (y^2 + \gamma_n^2) \cos(\gamma_n) = 0$$
where $(2n-)\ (\pi/2) < \gamma_n < n\pi, n \geq 1$ (58)
shows relationship of quantities

Equations 54 through 56 are expressed in terms of three independent dimensionless parameters, ß, t_s, and s. Neuman (1975) decreased the number of independent dimensionless parameters by considering the case in which $s = S/S_y$ approaches zero, that is, in which S is much less than S_y. The results are two asymptotic families of type curves referred to as type A and type B curves (Figure 18). Neuman (1975) listed numerical values for the curves.

The curves lying to the left of the values of ß in Figure 18 are called type A curves and correspond to the top scale expressed in terms of t_s. The curves lying to the right of the values of ß in Figure 18 are called type B curves and correspond to the bottom scale expressed in terms of t_y. The two sets of curves are asymptotic to Theis curves. Type A curves are intended for use with early drawdown data and type B curves with late drawdown data.

Application:

Neuman (1975) described application of his solution for aquifer characteristics by two methods: Using logarithmic plots of aquifer test data and type curves, and using semilogarithmic plots of aquifer test data.

The logarithmic method as described by Neuman (1975) follows. Late-time drawdown, s, is plotted for the observation well on logarithmic tracing paper against values of time, t. This data plot is overlain on the type B curves; while keeping the vertical and horizontal axes of both graphs parallel, as much of the late time drawdown data is matched to a particular curve as possible and a match point is selected. The value of ß of the type curve matched is noted and the coordinates of s, s_D, and t, t_y of the match point are recorded. The transmissivity is calculated from

$$T = \frac{(QsD/s)}{4\pi}$$ (59)

and the specific yield from

$$S_y = \frac{Tt}{r^2 t_y}$$ (60)

Next, the process is repeated by overlaying the early time drawdown data on the type A curves. The value of ß corresponding to the type
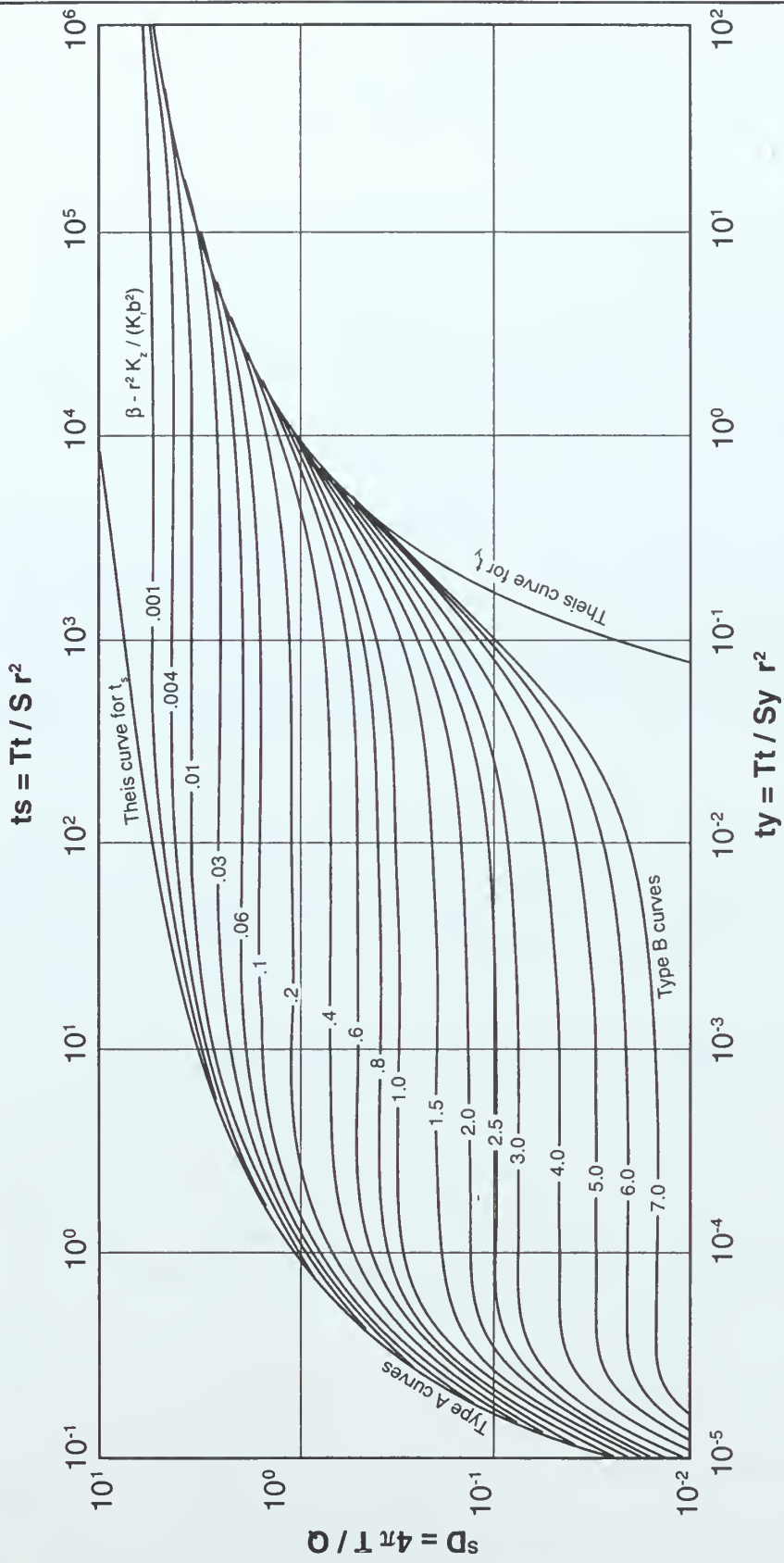
Figure 18. Type curves for fully penetrating wells in an unconfined aquifer (After Neuman, 1975).

curve must be the same as that obtained earlier from the B curves. The coordinates of the new match point s, s_D, and t, t_S are recorded. The transmissivity is again calculated from equation 59. Its value should be approximately equal to the previously calculated value from the late-time drawdown data as discussed by Bedinger and others (1988). The storage coefficient is calculated from:

$$S = \frac{Tt}{r^2 t_s} \tag{61}$$

The horizontal hydraulic conductivity, K_r, is calculated from the value of ß according to:

$$K_r = \frac{T}{b} \tag{62}$$

The degree of anisotropy, K_D is calculated from:

$$K_D = \frac{\beta b^2}{r^2} \tag{63}$$

The vertical hydraulic conductivity, K_z, is calculated from:

$$K_s = K_D K_r \tag{64}$$

Utilization of Confined Aquifer Methods to Unconfined Aquifers

The methods of Theis (1935) and Theim (1906), and other methods, though applicable to confined aquifers, may also be applied to unconfined aquifers where the drawdown is small in relation to the thickness of the aquifer (Jacob, 1950). Corrections in drawdown need to be made when the drawdown is a significant fraction of the aquifer thickness. Such corrections are usually called thin-aquifer corrections. These methods rely on the Dupuit-Forcheimer assumptions and are not valid for early time when vertical flow components are substantial. The Dupuit-Forcheimer assumptions state that, within the cone of depression of a pumping well, the head is constant throughout any vertical line through the aquifer and is, therefore, represented by the water table as discussed by Kruseman and Ridder (1991). Actually, this is true only in a confined aquifer having uniform hydraulic conductivity and a fully penetrating pumping well. Jacob (1963a) stated that where the drawdown needs to be replaced by s', the drawdown that would occur in an equivalent confined aquifer would be represented by

$$s' = s - \left(\frac{s^2}{2b}\right) \tag{65}$$

Jacob (1963a) presented a correction for the coefficient of storage where the drawdown is a substantial fraction of the original saturated thickness,

where

$$S' = (\frac{(b-s)}{b}) S \tag{66}$$

Neuman (1975) recommended that the use of confined aquifer methods for unconfined aquifers be restricted to late time data after the effect of delayed gravity response.

Where vertical components of head may be substantial, paired observation wells that partially penetrate the aquifer may be used in lieu of fully penetrating observation wells. One well of the pair is screened at the bottom of the aquifer and the other is screened just below the water table. The water levels in the paired wells are averaged and used in the confined aquifer method along with the thin aquifer correction, as necessary (Lohman, 1972).

Estimating Stream Depletion by Pumping Wells

The correlation between stresses imposed by pumping wells and the resultant depletion of stream flows has been identified by numerous investigators (Glover and Balmer, 1960; Theis and Conover, 1963; Hantush, 1964). This correlation is usually shown by charts and equations as discussed by Jenkins (1970). The techniques shown in this section are mainly derived from the work of Jenkins (1970) who provided easy to follow tools such as curves, tables, and sample computations. The symbols that are employed in this section are defined below:

T	=	transmissivity $[L^2/T]$
S	=	specific storage of the aquifer, dimensionless
t	=	time, during pumping period, since pumping began $[T]$
tp	=	total time of pumping
ti	=	time after pumping stops $[T]$
Q	=	net steady pumping rate $[L^3/T]$
q	=	rate of depletion of the stream $[L^3/T]$
Qt	=	net volume pumped during time t $[L^3]$
Qtp	=	net volume pumped $[L^3]$
v	=	volume of stream depletion during time t, tp + ti $[L^3]$
a	=	perpendicular distance from pumped well to stream $[L]$
sdf	=	stream depletion factor $[T]$.

Jenkins (1970) defines the stream depletion factor to be the time coordinate where the volume of stream depletion is equal to 28 percent of Qt on a curve relating v to t, and if sdf = a^2S/T. In a complex system, it can be considered to be an effective value of a^2S/T. Jenkins (1970) further states that the value of the sdf at any location in the system depends upon the integrated effects of the following: irregular impermeable

boundaries, stream meanders, aquifer properties and their areal varia-
tion, distance from the stream, and specific hydraulic connection be-
tween the stream and the aquifer. It should be noted that the curves
and tables used for calculating depletions in this section are dimension-
less and can be used with any units as long as they are consistent.
Jenkins (1970) states that the assumptions used in the analysis of calcu-
lating stream depletion from pumping wells are as follows:

1. T does not change with time.
2. The temperature of the stream is assumed to be constant and
   the same as the temperature of the aquifer.
3. The aquifer is isotropic, homogenous, and semi-infinite in
   areal extent.
4. The stream that forms the boundary is straight and fully pen-
   etrates the aquifer.
5. Water is released instantaneously from storage.
6. The well is open to the full saturated thickness of the aquifer.
7. The pumping rate is steady during any period of pumping.

Curves A and B in Figure 19 apply during the period of steady state
pumping as discussed by Jenkins (1970). Curve A defines the correla-
tion between the dimensionless term, t/sdf, and the rate of stream deple-
tion, q, at time t, and is shown as a ratio to the pumping rate Q. Curve
B defines the correlation between t/sdf and the volume of the stream
depletion, v, during time t, and is defined as a ratio to the volume
pumped, Qt. The curves 1-q/Q and 1-v/Qt are defined to better interpret
values of q/Q and v/Qt when the ratios surpass 0.5. The coordinates of
curves A and B are tabulated in Table 2. The curves A and B that are
tabulated in this section are after Jenkins (1970). It should be noted
that the precision is only to two significant places, which is considered to
be appropriate for this type of analysis.

Sample Problem

To explain the application of the curves and table, a sample problem is
defined and solved using the method outlined in this section. The prob-
lem is typical of what may be encountered in the field or as a proposed
activity. It is assumed that the data used in the examples is usually
available during the field study phases of most water resource projects.

The problem is a pumping well that is pumped at 2.0 acre feet per day
and is located 1.58 miles from a stream. The question is: How long can
the pumping continue before the stream depletion reaches 0.14 acre feet
per day, and what is the total stream depletion for the period of pumping?
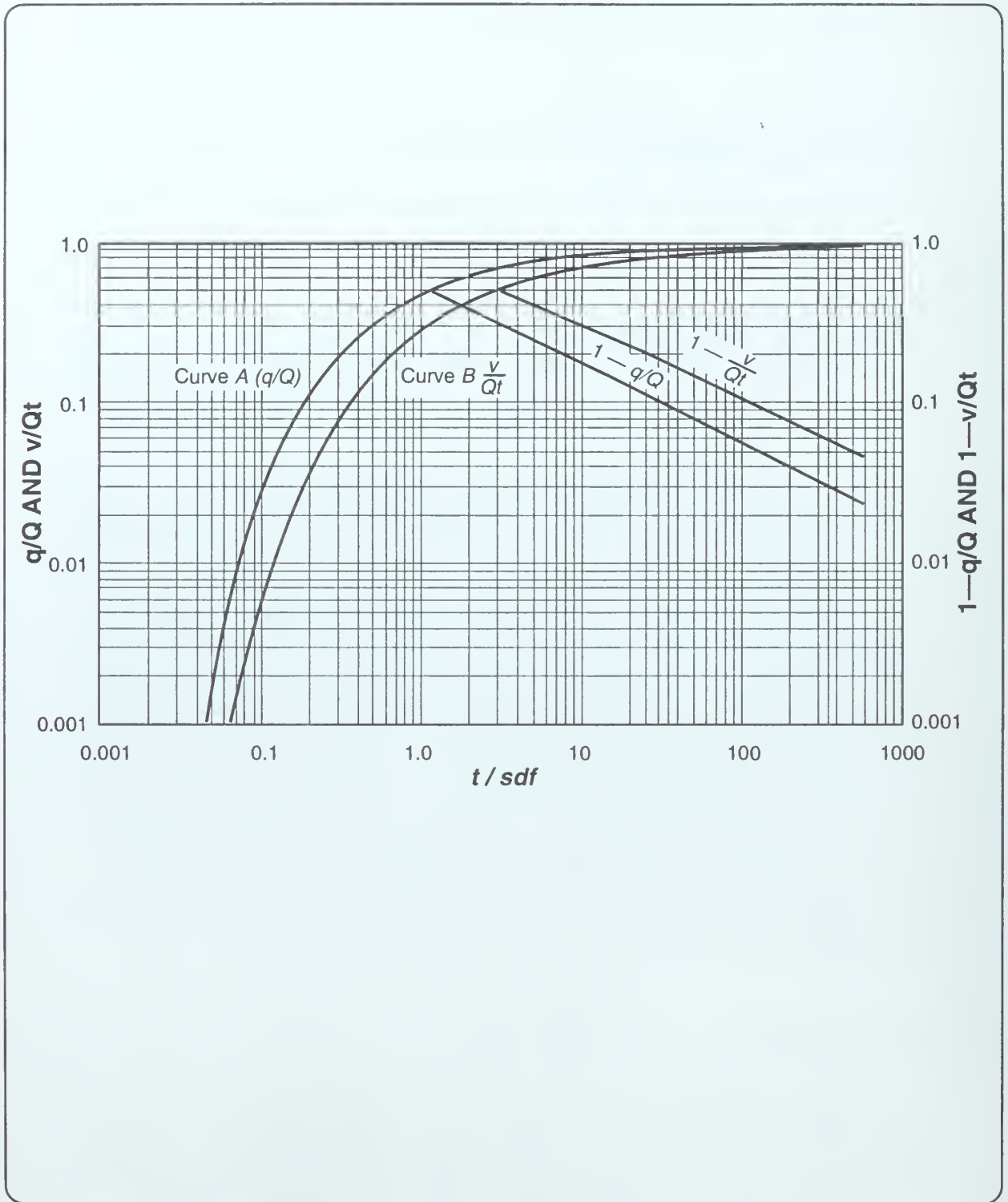
Figure 19. Curves to interpret rate and volume of stream depletion
(After Jenkins, 1970).

54

Table 2.   Values of q/Q, Q, v/Qt, and v/Qsdf relating selected values of t/sdf

Values of q/Q, v/Qt, and v/Qsdf corresponding to selected values of t/sdf

$\dfrac{t}{sdf}$	$\dfrac{q}{Q}$	$\dfrac{v}{Qt}$	$\dfrac{v}{Qsdf}$
0	0	0	0
.07	.008	.001	.0001
.10	.025	.006	.0006
.15	.068	.019	.003
.20	.114	.037	.007
.25	.157	.057	.014
.30	.197	.077	.023
.35	.232	.097	.034
.40	.264	.115	.046
.45	.292	.134	.060
.50	.317	.151	.076
.55	.340	.167	.092
.60	.361	.182	.109
.65	.380	.197	.128
.70	.398	.211	.148
.75	.414	.224	.168
.80	.429	.236	.189
.85	.443	.248	.211
.90	.456	.259	.233
.95	.468	.270	.256
1.0	.480	.280	.280
1.1	.500	.299	.329
1.2	.519	.316	.379
1.3	.535	.333	.433
1.4	.550	.348	.487
1.5	.564	.362	.543
1.6	.576	.375	.600
1.7	.588	.387	.658
1.8	.598	.398	.716
1.9	.608	.409	.777
2.0	.617	.419	.838
2.2	.634	.438	.964
2.4	.648	.455	1.09
2.6	.661	.470	1.22
2.8	.673	.484	1.36
3.0	.683	.497	1.49
3.5	.705	.525	1.84
4.0	.724	.549	2.20
4.5	.739	.569	2.56
5.0	.752	.587	2.94
5.5	.763	.603	3.32
6.0	.773	.616	3.70
7	.789	.640	4.48
8	.803	.659	5.27
9	.814	.676	6.08
10	.823	.690	6.90
15	.855	.740	11.1
20	.874	.772	15.4
30	.897	.810	24.3
50	.920	.850	42.5
100	.944	.892	89.2
600	.977	.955	573

(After Jenkins, 1970)

Data

$$q = 0.14 \text{ ac-ft/d}$$
$$Q = 2.0 \text{ ac-ft/d}$$
$$a = 1.58 \text{ miles}$$
$$T/S = 106 \text{ gal/d/ft}$$
$$t1 = 30 \text{ days}$$
$$sdf = a^2S/T = a^2/(T/S) =$$
$$(1.58\text{mi})^2 (5,280 \text{ ft/mi})^2 / 10^6 \text{ gal/d/ft}) (1 \text{ ft}^3/7.48 \text{ gal}) =$$
$$520 \text{ days}$$

Find

tp
v at tp
q at tp + ti
v at tp + ti
q max
t of q max

Solution

From the data given, the ratio of the rate of stream depletion to the rate of pumping is $q/Q = (0.14\text{ac-ft/d})/(2.0 \text{ ac-ft/d}) = 0.07$

From curve A (Figure 19) t/sdf = 0.15

Substitute the value under Data for sdf, and t = (0.15) (520 days) = 78 days

The total time the well can be pumped is 78 days

When t/sdf = 0.15 then from curve B (Figure 19) v/Qt = 0.02

Substitute the values for Q and t, and the volume of the stream depletion during this time period is v = (0.02) (2ac-ft/d) (78 days) = 3.1 ac-ft

This shows that during the 78-day pumping period, 3.1 ac-ft of water can be attributed as stream depletion.

It should be noted that variation from idealized stream conditions may cause actual stream depletions to be either more or less than the values interpreted from the method discussed in this section. Fluctuations in water temperature will cause variations in stream depletion, particularly by large-capacity wells near stream lengths. As discussed by Moore and

Jenkins (1966), if large-capacity wells are located close to a stream length and streambed permeability is low compared to aquifer permeability, the water table may be drawn down below the bottom of the streambed. The methods discussed in this section are not appropriate for streambed permeability, area of the streambed, temperature of the water, and stage of the stream.

The mathematical basis for the curve development and table presented in this section is beyond the scope of this guide. If the reader is interested in a more detailed discussion of the mathematical curve and table development, they are referred to the work of Glover (1954), Jenkins (1968a), Theis (1941), and Theis and Conover (1963).

Pumping Well and Flow System Characteristics

Pumping Well Conditions

The aquifer test methods discussed in the previous sections are, for the most part, based on simple geometric conditions and constant discharge or head change in the pumping well. For example, in the methods for confined aquifers, the pumping well was assumed to fully penetrate the aquifer and result in radial flow and vertically uniform heads in the aquifer. In the methods for unconfined aquifers, the pumping well and the observation well were assumed to be fully penetrating, which simplifies the analytical solution and application of the method to the problem. Storage effects in the pumping well were assumed to be negligible, except in the slug test methods. Variations from simple geometric conditions of the pumping well and variable discharge may cause anomalous hydraulic conditions in the flow field as discussed by Bedinger and others (1988). Disregard of pumping well geometry, that is, such factors as length of screened interval and depth of screen, or in some cases, of observation well geometry, may invalidate an aquifer test method. In this section, some principal methods are discussed for accounting for partially penetrating wells, variations in the discharge rate of the pumping well, constant drawdown, storage in the pumping well, inertial effects of water in the pumping well and in the aquifer, multiple aquifers, fractured media, anisotropic media, and image wells.

Partially Penetrating Wells

Partially penetrating pumping wells cause vertical components of flow that greatly complicate the analytical solution to the hydraulic head distribution in the aquifer and the application of the solution to aquifer tests. For these reasons, methods of confined aquifer test analyses treated previously assume fully penetrating pumping wells and radial flow. The analytical method of Neuman (1972 and 1975) for unconfined aquifers presented in the section on *Hydraulic Test Methods for Aquifers* is based on the assumptions of completely penetrating pumping and observation wells. These assumptions made it possible to simplify the application of the method using a single family of type curves.

The vertical components of head caused by partial penetration of the pumping well need to be considered for $(K_z/K_r)^{1/2}$ r/b < 1.5 (Reed, 1980). Thus, in a homogeneous, isotropic confined aquifer, where $K_z = K_r$, the effects of a partially penetrating pumping well are negligible beyond a

distance of about 1.5 times the aquifer thickness as discussed by Bedinger and others (1988). In an aquifer having radial-vertical anisotropy, where $K_z > K_r$, vertical flow components are of concern for a greater distance from the pumping well. Analytical solutions have been presented for anisotropic confined and unconfined aquifers and methods have been developed for their application to aquifer tests. These methods are discussed in the section on *Anisotropic Aquifer Materials*.

Variably Discharging Wells

Stallman (1962) and Moench (1971) presented methods of analysis of drawdown in response to an arbitrary discharge function. These methods simulate pumpage as a sequence of constant rate step changes in discharge. The methods utilize the principle of superposition in construction type curves by summing the effects of successive changes in discharge. The type curves may be derived for pumping wells discharging from extensive, leaky, and nonleaky confined aquifers, or any situation where the response to a unit stress is known.

Recognizing that the uncontrolled discharge from a pumping well commonly decreases with time during the early period of pumping, type curves have been described for drawdown in response to decreasing discharge functions that can be expressed mathematically. Hantush (1964b) developed drawdown formulas for three types of decrease in pumping well discharge including an exponentially decreasing discharge and a hyperbolically decreasing discharge for extensive, uniform confined aquifers. Methods for leaky, sloping leaky, and nonleaky confined aquifers also were presented by Hantush (1964b).

Abu-Zied and Scott (1963) presented a general solution for drawdown in an extensive confined aquifer in which the discharge of the pumping well decreases at an exponential rate. Aron and Scott (1965) proposed an approximate method of determining transmissivity and storage from an aquifer test in which discharge decreases with time during the early part of the test.

Lai and Su (1974) presented methods for determining the drawdown in a homogeneous, isotropic, nonleaky, confined aquifer, taking into account storage in the pumping well in response to exponentially and linearly decreasing discharge. Lai and Su (1974) also presented a method for determining drawdown in a homogeneous, isotropic, leaky confined aquifer, taking into account storage in the pumping well and various discharge rates.

Constant Drawdown Conditions

Methods to determine the hydraulic head distribution around a pumping well in a confined aquifer with near constant drawdown are presented by Jacob and Lohman (1952), Hantush (1964b), and Rushton and Rathod (1980). Such conditions are most commonly achieved by shutting in a flowing well long enough for the head to fully recover, then opening the well. The solutions of Jacob and Lohman (1952), and Hantush (1964a) apply to areally extensive, nonleaky confined aquifers. Rushton and Rathod (1980) used a numerical model to analyze aquifer test data. When using the method of Jacob and Lohman (1952), measurements are made of the decreasing rate of flow after the pumping well is opened. Application of the method by type curve and straight-line techniques is described by Lohman (1972). Hantush (1959b) presented two methods for determining constant drawdown in a leaky confined aquifer without storage in the confining beds. One method is for the discharge of the pumping well; the other is for the drawdown in the aquifer. Reed (1980) presented a computer program for calculating function values for Hantush's (1960) methods. The method of Hantush (1964a) uses measurements of head in the flowing pumping well and in an observation well to determine diffusivity (T/S).

Storage and Inertial Influence

The effect of storage and inertial effects of head in the pumping well and the aquifer were examined by Bredehoeft and others (1966). For continuous pumping under ordinary conditions of pumping from production wells in transmissive aquifers, the effects of storage in a production well become negligible in a short time. However, the effects of storage could be significant in pumping wells of large diameter drawing water from aquifers having minimal transmissivity as discussed by Bedinger and others (1988). The effect of slug well storage is commonly significant in slug tests and the slug test methods presented in an earlier section of this guide account for storage in the slug well. Most aquifer test methods do not consider the effect of storage within the pumping well; hence, the stated assumption that the pumping well is of infinitesimal diameter. According to Papadopulos and Cooper (1967), the pumping well storage may be neglected if $t > 2.5 \times 10^2 r_c^2/T$, where r_c is the radius of the well casing in the interval in which the water level declines.

Papadopulos (1967b) presented a solution for determining the drawdown in and around a pumping well of finite diameter taking into consideration the effect of water stored in the wellbore. Papadopulos (1967b) presented tables and type curves and discussed application of type curve techniques for solution(s) to the problem. Tables and type curves for application of the method are presented in Reed (1980).

Inertial effects in a well are a function of the well and the aquifer. Force-free oscillation occurs in underdamped wells following events such as earthquakes or sudden imposition of a head change. Bredehoeft and others (1966) presented examples in which the column of water in underdamped wells oscillates for a few seconds after a sudden commencement of continuous pumping. Inertial effects to continuous pumping are probably not significant in most aquifer tests.

Van der Kamp (1976) and Kipp (1985) presented methods for determining the transmissivity of an aquifer from inertially induced oscillation in a pumping well, a response that may occur in conjunction with extremely transmissive aquifers. Van der Kamp (1976) suggested a technique for inducing oscillations in a well by a procedure used in some slug tests; that is, by sudden removal of a closed cylinder of known volume for the well. Kipp (1985) presented the complete method from the noninertially induced slug well response to the freely oscillating slug well. The method of Kipp (1985) is a useful extension to conventional slug test methods. Slug test methods are suitable for damped slug wells, those in which force-free oscillations are negligible as is common in aquifers with minimal to average transmissivity.

Flow System Characteristics

Flow system characteristics for the aquifer test methods discussed in the previous sections were based on simple geohydrologic characteristics. Aquifers were assumed to be homogeneous and isotropic and of infinite areal extent. In this section, methods are introduced which deal with multiple, fractured, and anisotropic aquifers, and with aquifers of finite areal extent bounded by impermeable and constant head boundaries as discussed by Bedinger and others (1988).

Multiple Aquifers

Tests of multiple aquifers, that is, two or more aquifers separated by a leaky confining bed or penetrated by a pumping well, require special methods for analysis. Bennett and Patten (1962) devised a method for testing a multiaquifer system by a procedure using downhole metering and constant drawdown. Extending his work with leaky aquifers, Hantush (1967b) presented a solution for determining drawdown distribution in two aquifers separated by a leaky confining bed, in which storage is neglected, in response to discharge from one or both of the aquifers. Neuman (1972) provides a solution for drawdown in leaky confining beds above and below an aquifer being pumped. Neuman and Witherspoon (1969a) developed an analytical solution for the flow in a

leaky confined system of two aquifers separated by a leaky confining bed with storage. One of the aquifers is discharged through a fully penetrating well. Javandel and Witherspoon (1969) presented a finite element method of analyzing anisotropic multiaquifer systems.

Fracture Flow

Models that have been developed for flow in fractured rock include those based on the assumptions that flow is in a single fracture composed of parallel plates, flow is in a network of intersecting fractures, and flow is in a double porosity medium consisting of blocks containing intergranular porosity and permeability, the blocks being separated by a network of intersecting fractures sufficiently extensive to be considered a continuum. A review of methods of treating fractured media is presented in Gringarten (1982).

Solution for flow in single finite fractures in a porous medium is presented by Gringarten and Ramey (1974). Barenblatt and others (1960) presented a method for solving the double porosity model. This model is based on the assumptions that storage of water in the fractures is negligible compared to storage in the pores of the blocks, and flow of water is primarily in the fractures. Boulton and Streltsova (1977) presented a solution for a system composed of porous layers separated by fractured layers that are horizontal. Moench (1984) developed type curves for a double porosity model with a fracture skin that may be present at the fracture block interfaces as a result of mineral deposition or alteration.

Anisotropic Aquifer Materials

Most of the aquifer test methods discussed in the section on *Hydraulic Test Methods for Aquifers* are based on the assumption that the aquifer is homogeneous and isotropic. Natural materials are neither homogeneous nor isotropic, but aquifer test methods based on this assumption are widely applied and useful. Aquifer test methods and associated procedures that have been devised to evaluate anisotropy of natural media will be introduced in this section. Vertical anisotropy is common in stratified sediments. Anisotropy often is characteristic of natural formations. Hantush (1966a and 1966b) presented methods for determining flow in homogeneous, anisotropic media, but did not provide procedures for applying the methods. Methods described in the literature for treating anisotropy are limited to the situations of either horizontal anisotropy or horizontal and vertical anisotropy.

Solutions to the head distribution in a homogeneous confined aquifer with radial vertical anisotropy in response to constant discharge of a partially penetrating well are presented by Hantush (1961a).
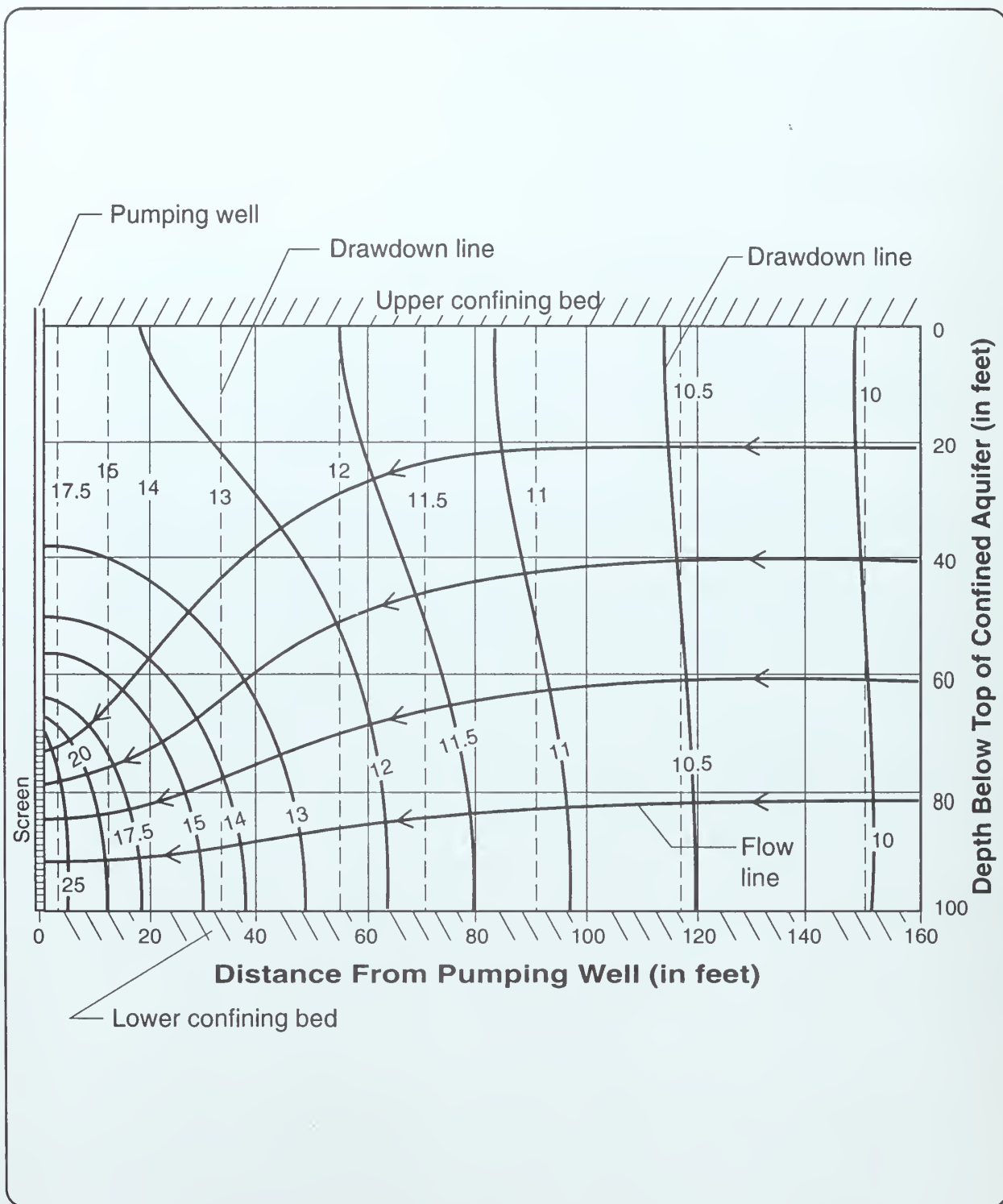
Figure 20. Section showing drawdown and flow paths near a pumping well in an ideal nonleaky confined aquifer. Solid lines are drawdown and flow lines for a pumping well screen in bottom of aquifer; dashed lines are drawdown lines for a pumping well screened the full aquifer thickness (After Weeks, 1969).

The solutions of Hantush (1961a) were applied by Weeks (1964 and 1969), who presented methods to determine the ratio of horizontal to vertical hydraulic conductivity. The analyses are made by comparing measure drawdowns in the piezometers to those predicted if the pumping well fully penetrates the aquifer. The differences in the measured and predicted drawdowns are determined, and the distances from the partially penetrating pumping well at which these differences would occur in an isotropic aquifer are determined from an equation. The permeability ratio is computed as the square of the ratio of the actual distances to the computed distances. Weeks (1969) applied graphical methods to the solution of vertical and horizontal hydraulic conductivity and presented tables of values of the dimensionless drawdown correction factor (Figure 20). Weeks (1969) also discussed conditions for which his method is applicable to unconfined aquifers.

Papadopulos (1965) presented a method for determination of horizontal plane anisotropy in an areally extensive, homogeneous, confined aquifer. Papadopulos (1965) introduced a graphical method for solution of the components of the transmissivity tensor from aquifer test data using a minimum of three observation wells. Hantush and Thomas (1966) presented a graphical method of determining horizontal anisotropy in confined aquifers from the elliptical shape of the cone of drawdown.

Neuman (1975) presented a solution for the drawdown in piezometers in response to discharge from a partially penetrating pumping well in an unconfined aquifer having radial and vertical anisotropy. Because of the large number of variables involved, Neuman (1975) offered to provide a computer program from which the user could prepare type curves for specific cases.

Image Well Method

Each of the aquifer test methods of aquifer tests discussed previously in this guide is based on the assumption that the aquifer is of infinite areal extent. It is recognized that such conditions do not exist. Effects of limitations in areal extent of aquifers by impermeable boundaries or by source boundaries, such as hydraulically connected streams, may preclude the direct application of an aquifer test method. The method of images provides a tool by which a solution to the problem of exterior boundaries can be devised as discussed by Bedinger and others (1988). This method uses the substitution of a hydraulic boundary for the physical feature.

Consider first an aquifer bounded by a perennial stream in which the head is independent of the pumping well; that is, there is no drawdown

65
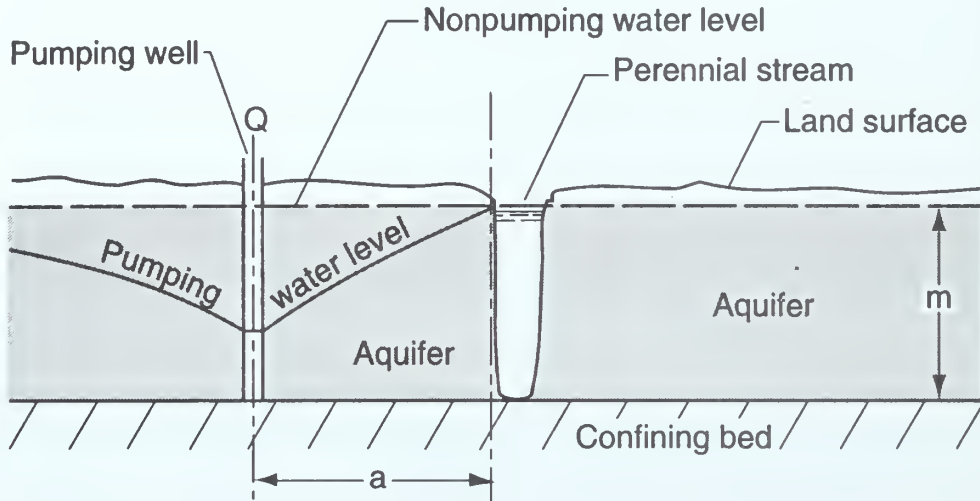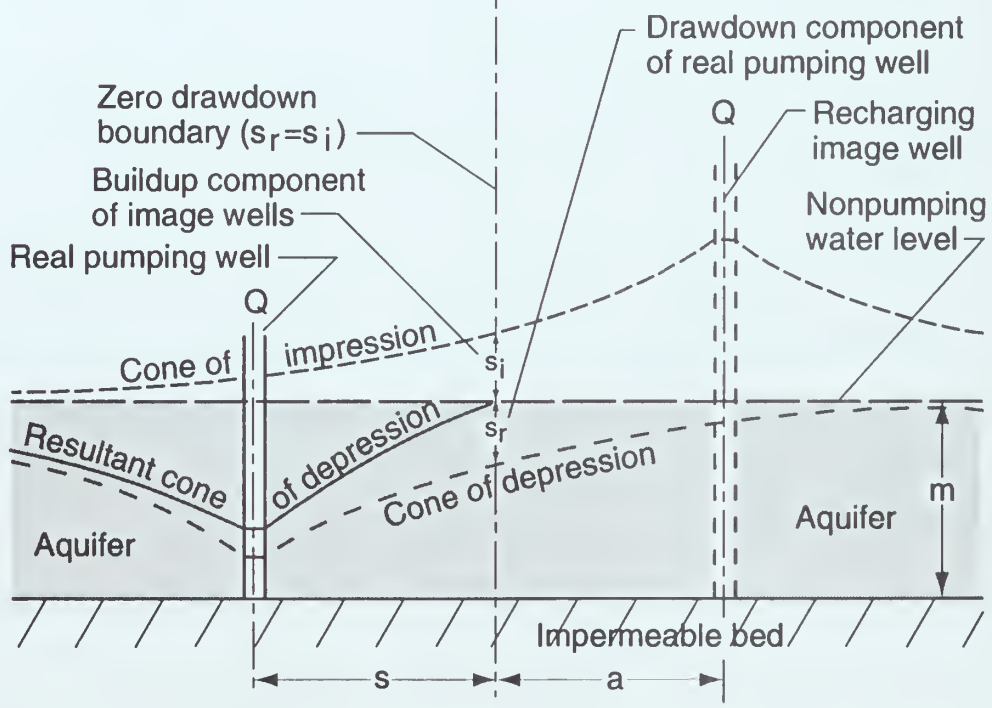
Figure 21. Idealized sections of a pumping well in a semi-infinite aquifer bounded by a perennial stream and of the equivalent hydraulic system in an infinite aquifer (After Ferris and others, 1962).

in the stream and the stream functions as a fully penetrating, constant head boundary to the aquifer (Figure 21A). An image system that satisfies the foregoing boundary condition is shown in Figure 21B; that is, an imaginary recharging well located on the opposite side of and the same distance from the stream as the real pumping well. Both wells are on a line perpendicular to the stream. The imaginary recharge well operates simultaneously with the real pumping well and recharges water to the system at the same rate the real well discharges. The resultant drawdown at any point in the system is the algebraic sum of the drawdown caused by the real well and the rise in water level caused by the imaginary wells.

Next, consider an aquifer bounded by confining material (Figure 22A). The hydraulic boundary condition imposed by the confining material is that there is no flow across the material. The image well condition that duplicates this physical condition by hydraulic analogy is shown in Figure 22B. An imaginary pumping well has been placed at the same distance from the line of zero flow as the real well. The wells are on the opposite sides of and on a line perpendicular to the line of zero flow as discussed by Bedinger and others (1988). As in the case of the flow system with the recharging image well, the resultant drawdown at any point in the system is the algebraic sum of the changes in head caused by the real and imaginary well.

The theory of images may be applied to any combination of straight-line constant head and impermeable boundaries. A number of combinations are discussed by Ferris and others (1962). Because the drawdown in an observation well, s_0, in a system bounded by a line source or impermeable boundary, is the algebraic sum of the components of drawdown by the pumping well, s_p, and by the image well, s_i, the hydraulic head distribution in the aquifer can be analyzed by superposing the solutions, by an appropriate aquifer test method for the flow system, for the real and image wells. For example, if the flow system is a confined nonleaky aquifer, it would be appropriate to apply the method of Theis in equation 4. The Theis equation for an aquifer bounded by line source or impermeable boundary where

$$s_0 = s_p \pm s_i \tag{67}$$

becomes (Stallman, 1963)

$$s_0 = \frac{Q}{4\pi T} [W(u) \, p \pm W(u) \, i]$$

$$= \frac{Q}{4\pi T} \sum W(u) \tag{68}$$

67
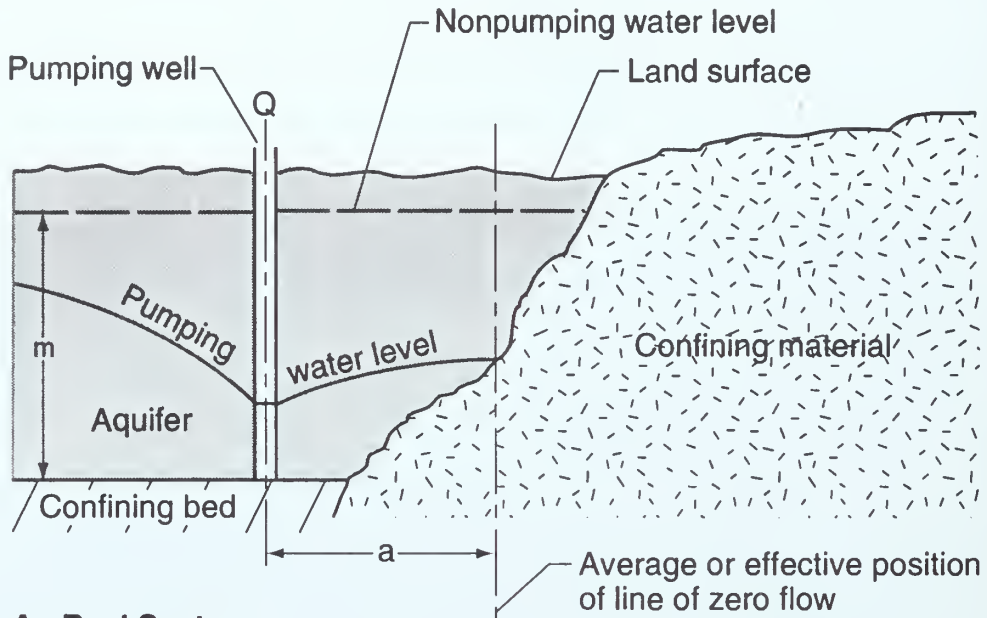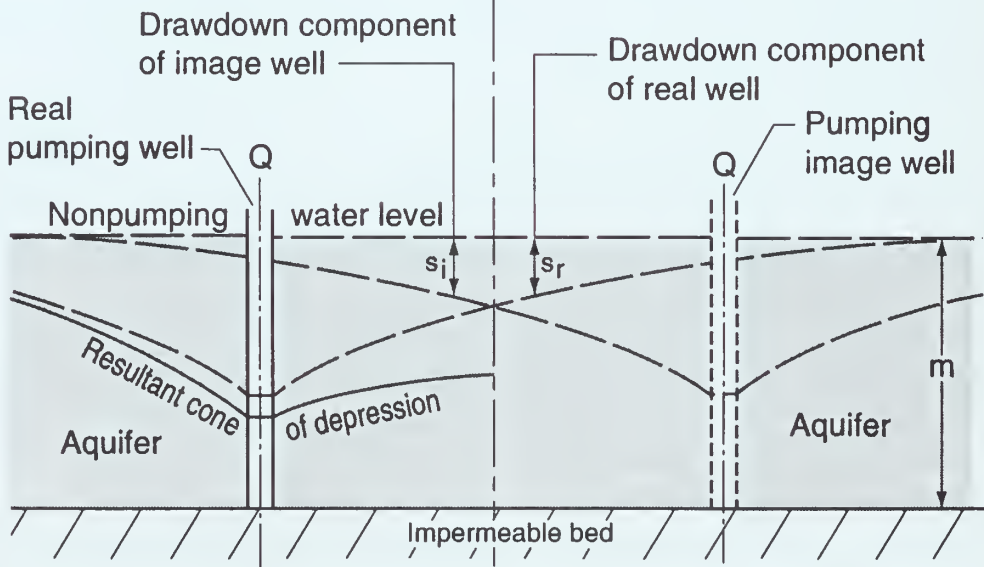
A. Real System

B. Hydraulic Counterpart of Real System

Figure 22. Idealized sections of a pumping well in a semi-infinite aquifer bounded by an impermeable formation and of the equivalent hydraulic system in an infinite aquifer (After Ferris and others, 1962).

and

$$u_p = \frac{r_p^2 S}{4Tt} \tag{69}$$

and

$$u_i = \frac{r_i^2 S}{4Tt} \tag{70}$$

Type curves can be constructed for a specific observation well or a family of type curves can be drawn for different ratios of $r_i = r_p$. Such a family of type curves is presented by Stallman (1963) and Lohman (1972), who discuss application of the method. The type curves can be used to analyze the drawdown data for hydraulic properties of the aquifer in a system where a boundary is known to occur or to locate the position of a hidden boundary that is indicated by the drawdown data from an aquifer test (Morris and others, 1959; Moulder, 1963). Boundaries in nature may be neither absolutely impermeable nor constant head. For example, streams generally do not fully penetrate the aquifer and streambed materials may limit the rate of water movement from the stream to the aquifer.

Methods to determine an effective distance from a pumped well to a stream boundary include a type curve method suggested by Kazman (1946) that is implicit in the type curves of Stallman (1963) and Lohman (1972), and in the graphical extrapolation of the drawdown to zero from the water level in a line of observation wells perpendicular to the river from the pumped well of Rorabaugh (1956).

Summary

Geohydrology is an important part of all ground water resource projects with respect to analysis and design. Water resource projects include the modeling, planning, analysis, and interpretation of information on the subsurface environment of ground water. Critical elements of the data collection phases of a ground water resource project may be closely associated with geohydrologic testing.

The expanding scientific literature on ground water hydraulics and hydrology should be read and evaluated on a continuing basis. The information contained in this guide will need to be supplemented as more updated methodologies and techniques become available.

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Appendix A
Summary of Hydraulic Test Methods

This summary includes methods for hydraulic testing classified by aquifer condition, pumping well characteristics, recharge and discharge function, and boundary conditions. The summary is divided into three parts: Confined Aquifer, Unconfined Aquifer, and Other Conditions.

I.   Confined Aquifer

    A.   Nonleaky confined aquifer

       Methods included here are for radial flow in a nonleaky, porous, homogeneous, and isotropic medium of infinite areal extent. Change in water stored is instantaneous and proportional to the change in head. The aquifer is confined above and below by impermeable beds. The water level is above the top of the aquifer.

       1.   Constant flux.

          Theim (1906) - Asymptotic (pseudosteady) solution

          Theis (1935) - Negligible storage in pumped well

          Cooper and Jacob (1946) - Asymptotic (logarithmic) approximation to well function of Theis (1935) with increasing time and decreasing radial distance.

          Stallman (1963) - Aquifer bounded on one side by a straight boundary (either constant head or no flow.)

       2.   Constant drawdown.

          Jacob and Lohman (1952) - Step change in water level at the pumping well. Discharge of the pumping well as a function of time. Commonly used for shut-in flowing pumping wells.

3.   Instantaneous head change (slug tests).

A rapid change in water level is induced in the slug well by various methods, such as injection, bailing, or pressurization. Inertial effects are assumed to be negligible. Inertially induced oscillatory fluctuations and applications to slug tests are treated by Krauss (1974), Van der Kamp (1976), Shinohara and Ramey (1979), and Kipp (1985).

Hvorslev (1951) - Applies differential equation for permeameters to head change in an aquifer. Cases involving both radial and vertical flow are treated by shape factors for different flow geometries.

Skibitzke (1958) - A method for determining the water level in a well after it has been bailed. Bailed well is assumed to be a fully penetrating line source rather than a well of finite diameter.

Ferris and Knowles (1963) - Change in water level is caused by a sudden injection of water. Injection well is assumed to be a fully penetrating line source rather than a well of finite diameter.

Cooper and others (1963) - Derives equation, presents curves and a table of functions, presents a method of determining transmissivity and storage coefficient taking into account well storage, and discusses the relation to the solution of Ferris and Knowles (1963). Can be applied to fractured rock (Wang and others, 1977) if fracture openings do not change with pressure and there is negligible drainage form the matrix into the fractures.

Papadopulos and others (1973) - Presents additional function values and curves for the method of Cooper and others (1967).

Bredehoeft and Papadopulos (1980) - Discusses testing formations with minimal permeability by pressurizing a shut-in well. For a certain range of parameter values, the method of Cooper and others (1967) indicates only the product of transmissivity and storage.

Neuzil (1982) - Discusses changes in procedure and equipment for the method of Bredehoeft and Papadopuos (1980).

Barker and Black (1983) - Considers an aquifer with uniform horizontal fissures, horizontal flow in the fissures, vertical flow in the matrix, and storage in both. Numerical inversions are used for analysis of errors resulting from matching slug test data form a fissured aquifer by the method of Cooper and others (1967), which was developed for a homogeneous aquifer. It was concluded that transmissivity will always be overestimated, although not likely by more than a factor of 3. Storage coefficient, however, could be several orders of magnitude either larger or smaller.

Dougherty and Babu (1984) - Application of slug tests to fractured porous aquifers.

4.   Variable discharge.

Flux of pumping well is not constant, but varies with time.

Werner (1946) - Flux is a linear function of time.

Stallman (1962) - Continuously varying discharge is approximated by step changes. Function curves for drawdown are sums of well function (Theis, 1935) weighted by the change in discharge.

Abu-Zied and Scott (1963) - Flux exponentially changes with time.

Abu-Zied and others (1964) - Treats special cases that simplify the method of Abu-Zied and Scott (1963).

Aron and Scott (1965) - Superposes the log asymptote to the solution of Theis (1935) weighted by the change in discharge.

Sternberg (1968) - Graphical summation based on the log approximation to the well function and a multiple-step approximation of the well discharge.

Moench (1971) - Convolution integral applied to a general discharge function and the method of Theis (1935). Representation of discharge as a step curve and evaluation of the integral by summation.

Lai and others (1973) - Includes the effect of storage in the well. Presents a general solution as a convolution integral. Presents solutions for exponential and linear discharge as unevaluated integrals of complex functions. No tables of values. Presents three type curves for linear decreasing flux of drawdown in the pumping well.

5.   Multiple Aquifers

Papadopulos (1966) - Two nonleaky aquifers, with different hydraulic properties, separated by a confining layer. Constant discharge from a well open to both aquifers and radial flow.

B.   Nonleaky, fractured, confined aquifers.

Fractures, rather than the medium, transmit most of the fluid, especially in the vicinity of the pumping well.

Gringarten (1982) - Review articles discuss several aspects of flow to wells through fractured media.

1.   Extensive fractures.

An extensive network of fractures, sufficiently dense and uniform as to be considered a continuum.

Barenblatt and others (1960) - Double porosity model. Fractures in a porous medium. All storage in the pores. Flow from the medium into the fractures is proportional to the difference in head. Solution for head in the fractures.

Warren and Root (1963) - Solutions to the conditions of Barenblatt and others (1960) that are applicable for long durations and for infinite and circular aquifers.

Boulton and Streltsova (1977) - Radial flow in fractures that are separated by uniform layers of

porous medium. Only vertical flow within the layers. Storage in fractures and layers. Methods for fractures and layers.

Dougherty and Babu (1984) - Analysis of slug tests in single and double-porosity aquifers by partially and fully penetrating wells with and without skin effect.

Moench (1984) - Double-porosity model with fracture skin at fracture-block interfaces.

Hsieh and others (1985) - Determination of three-dimensional, hydraulic-conductivity tensor in anisotropic fractured media.

2.   Single fracture.

A single fracture centered about the pumping well.

Gringarten and Ramey (1974) - Horizontal fracture.

Gringarten and others (1974) - Vertical fracture

C.   Nonleaky confined aquifer with radial flow and horizontal anisotropy.

Permeability and hydraulic conductivity are second order tensors in the horizontal plane. In two directions, the axes of the ellipse, flow, and hydraulic gradient are colinear. In other directions, flow and gradient are not parallel.

Papadopulos (1965) - A minimum of three observation wells at different directions from the pumping well are needed to determine the principal components and orientation of the transmissivity tensor.

Hantush (1966a) - Drawdowns measured in three lines of observation wells are needed. A line is one or more observation wells all in the same direction from the pumping well. If the principal directions are known, then two lines will permit analysis.

Hantush (1966b) - By a simple transformation of coordinates, the boundary-value problem describing the flow in a homogeneous media may be transformed to an equivalent

homogeneous and isotropic aquifer. Methods are discussed for leaky confined aquifers, for complete and partial penetration, and for decreasing discharge.

Hantush and Thomas (1966) - Distribution of observation wells is such that the elliptical shape of an equal drawdown (or residual drawdown for recovery) contour can be defined.

D. Leaky and confined aquifer with radial flow and isotropic and homogeneous porous media.

Vertical flow in uniform confining beds. Change in water stored is instantaneous with and proportional to change in head. Aquifer is confined above and below. Water level is above the top of the aquifer. Change in flow between aquifer and confining beds is proportional to drawdown.

1. Constant flux

Jacob (1946) - Solutions for steady flow in an extensive aquifer and nonsteady flow to a well at the center of a circular aquifer with no drawdown at the outer boundary.

Hantush and Jacob (1955) - Solution for nonsteady flow in an extensive aquifer.

Hantush (1956) - Graphical methods are applied to determine parameters.

Hantush (1960) - Solutions that apply at either short or long durations and that include the effect of storage in confining beds. The three combinations of zero drawdown of zero-flow boundaries above and below the system are presented.

Moench (1985) - Solution to transient flow to a well in an aquifer accounting for storage in the well and in the confining beds.

2.   Multiple aquifers.

Hantush (1967b) - Two aquifers separated by a con-
fining bed. No storage in the confining bed. Radial
flow in the aquifers and vertical leakage through the
confining bed is considered. Constant discharge from
one aquifer.

Neuman and Witherspoon (1969a) - Radial flow in
two aquifers separated by a confining bed. Vertical
flow is assumed only for the confining bed. Storage
in the confining bed is considered. Constant dis-
charge from an aquifer.

Neuman and Witherspoon (1972) - Design and analy-
sis of aquifer tests for leaky multiple aquifer sys-
tems. Observation wells in aquifer and confining bed
at same distance from pumping well.

3.   Constant drawdown.

Jacob and Lohman (1952) - Method uses measure-
ments of the decreasing flow rate after the well is
opened.

Hantush (1959b) - Solutions for drawdown away
from and discharge at the pumping well for exten-
sive, circular aquifers.

Rushton and Rathod (1980) - Analysis by numerical
methods.

4.   Variable discharge.

Hantush (1964b) - Solutions for drawdown corre-
sponding to three general types of decreasing dis-
charge.

Moench (1971) - Convolution integral applied to a
general discharge function and the method of
Hantush and Jacob (1955). Representation of dis-
charge as a step curve and evaluation of the integral
by summation.

Lai and Su (1974) - Includes the effects of storage within the pumping well. Presents a general method as a convolution integral. Presents methods for exponential, pulse function (pumping followed by recovery), and periodic (repeated pulse) discharge as unevaluated integrals of complex functions. No table of values. Presents 19 profiles of the cone of depression and 16 curves of drawdown in the pumping well as examples of the three discharge functions.

E. Nonleaky confined aquifer with homogeneous porous media and vertical flow components.

The pumping well, by the manner of its construction, is connected to only a part of the vertical extent of the aquifer. Consequently, vertical flow occurs in the vicinity of the pumping well.

Mansur and Dietrich (1965) - Discussion of a series of aquifer tests where a fully penetrating pumping well was backfilled to create successively smaller partial penetrations. Analysis of radial-vertical anisotropy through steady head distribution around the well. Head distribution determined both by electrolytic-analog model and using method of Muskat (1946).

Hantush (1964a) - Application to observation wells piezometers of the method derived by Hantush (1957). Tables of function values.

Hantush (1961d) - Methods of applying method of Hantush (1961a) to analysis of aquifer tests.

1. Entrance losses.

Jacob (1947) - Well loss is proportional to the square of the discharge.

Rorabaugh (1953) - Well loss is proportional to the n-th power of the discharge.

Lennox (1966) - Expresses formation loss as a function of time through the log approximation to the well function (Theis, 1935).

2. Inertial effects.

    Cooper and others (1965) - Response to seismic waves.

    Bredehoeft and others (1966) - Inertial and storage effects.
    Ramey (1979), and Kipp (1985).

    Krauss (1974), Shinohara and Ramey (1979), and Kipp
    (1985) - Provides solutions to the oscillatory fluctuations in
    a well after sudden injection or removal of a volume of
    water.

3. Storage effects.

    Papadopulos and Cooper (1967) - Drawdown in a large-
    diameter pumping well. Storage in the pumping well is an
    important factor in early response.

II. Unconfined Aquifer

    A.   Isotropic and homogeneous porous unconfined aquifer

          Boulton (1954a) - A radial, vertical, and time solution as-
          suming all storage at the water table. Most of the discus-
          sion and the limited number of function values are for
          drawdown at the water table.

          Boulton (1954b) - A radial and time method with storage
          throughout the aquifer. A source term in the differential
          equation is referred to as delayed yield or delayed drainage
          at the water table. The delayed yield drainage is the prod-
          uct of an empirical factor with the water-table storage and
          a convolution integral of rate of drawdown and an exponen-
          tial function.

          Boulton (1963) - Same conditions as in Boulton (1954b).
          Type curves and discussion of their use.

          Boulton (1964) - Reply in discussion of Boulton (1963).
          A short table of function values is included.

          Prickett (1965) - Use of the solution of Boulton (1963). Type
          curves and examples.

Stallman (1965) - Type curves constructed using analog
models for two cases where the pumping well is screened
throughout all of the bottom portion of the aquifer. Storage
only at the water table and vertical flow. Effects of radial-
vertical anisotropy are factored into the curves.

Norris and Fidler (1966) - An example of the use of the
method of Stallman (1965).

B.   Anisotropic unconfined aquifer.

Dagan (1967) - A partially penetrating pumping well. Stor-
age only at the water table. Anisotropy in the radial-
vertical plane by a change in scale.

Neuman (1972) - Fully penetrating pumping well. Storage
within the aquifer and at the water table. Anisotropy in
the radial-vertical plane.

Streltsova (1972) - An interpretation of the a of Boulton
(1955) as hydraulic conductivity (vertical direction) divided
by specific yield and a vertical length.

Neuman (1974) - Partially penetrating pumping well. Stor-
age within the aquifer and at the water table. Anisotropy
in the radial-vertical plane.

Neuman (1975) - Application of the methods of Neuman
(1972 and 1974). Tables and curves. Interpretation of the a
of Boulton (1954b and 1963) as not a constant, but varying
with radial distance from the center of the pumping well.

C.   Water table in confining layer overlying confined aquifer.

Cooley (1972) - Interpretation of the a of Boulton (1954b
and 1963) in terms of properties of an overlying confining
layer. Numerical models for this situation agree with the
method of Boulton (1954b).

Cooley and Case (1973) - The convolution integral of
Boulton (1954b and 1963) interpreted as the vertical
velocity at the base of a confining bed with negligible com-
pressibility. Numerical models indicated that the unsatur-
ated zone has little effect on flow in the aquifer.

Boulton and Streltsova (1975) - Partially penetrating pumping well. Storage within the aquifer and at the water table but not within the confining bed. Anisotropy in the radial-vertical plane. No curves or function values. Approximate solutions using method of Boulton (1963).

III. Other Conditions

Hantush (1962a) - Flow to a well in a nonleaky confined aquifer with a thickness that is an exponential function.

Hantush and Papadopulos (1962) - How to collect wells with lateral (horizontal) screens.

Bixel and others (1963) - Linear (half-plane) discontinuities in hydraulic conductivity or storage coefficient or both.

Brikowski (1993) - Estimating ground-water exchange between ponds or large-scale conduits embedded in uniform regional flow.

Moench and Prickett (1972) - Solutions for estimating movement of ground water from a pond or large scale radius conduit.

Zlottnik (1994) - using Dimensional analysis to determine and interpret slug test data in anistropic aquifers.

Appendix B
List of Nomenclature and Symbols

Symbol	Dimension	Description	Equations
A	L^2	Cross-sectional area	(2)
B	L	$(Tb'/K')1/2$	(32, 34, 38, 40)
B	l	Coefficient of head loss linearly related to the flow	(5)
C	1	Coefficient of head loss due to turbulent flow in the well, aquifer, and across the well screen	
D_0	L	$1.5b \backslash l \, r(,K_z/K_r)$	(4)
$F(\beta,\alpha)$	.....	F function of β,α	(28, 29)
H	L	Change in head in pumping/ slug well	(28)
H_0	L	Initial head rise in pumping/ slug well	(24, 28)
$H(u,\beta)$	.....	H function u,β	(49, 52)
J_0	.....	Zero order Bessel function of the first kind	(24, 27, 54)
J_1	.....	First-order Bessel function of the first kind	(24, 27)
K	LT^{-1}	Hydraulic conductivity of aquifer	(1, 2, 3, 37)
K'	LT^{-1}	Hydraulic conductivity of confining bed	(34, 36, 37, 40, 43, 44, 46, 48)
K'	LT^{-1}	Hydraulic conductivity of upper confining bed	(51)

K''	LT^{-1}	Hydraulic conductivity of lower confining bed	(51)
K_D	.....	Degree of anisotropy, equal to K_z/K_r	(63, 64)
K_r	LT^{-1}	Horizontal hydraulic conductivity	(4, 62, 64)
K_z	LT^{-1}	Vertical hydraulic conductivity	(4, 64)
$K_0(x)$	.....	Zero-order modified Bessel function of the second kind	(45, 47)
$L(u,v)$	.....	L (leakance) function of u,v	(35, 41)
Q	L^3LT^{-1}	Discharge rate	(2, 5, 6, 11, 13, 14, 15, 18, 19, 20, 22, 23, 32, 35, 38, 41, 45, 47, 49, 54, 59, 58)
S	.....	Storage coefficient	(7, 10, 13, 20, 23, 25, 31, 33, 35, 39, 42, 44, 50, 51, 61, 66, 69, 70, 71, 72)
S	.....	Storage coefficient	(7, 10, 12, 13, 20, 21, 25, 31, 33, 35, 39, 42, 44, 50, 51, 61, 66, 69, 70, 71, 72)
S'	.....	Storage coefficient of upper confining bed	(51)
S'	.....	Apparent coefficient of storage derived from use of corrected drawdowns	(66)
S''	.....	Storage coefficient of lower confining bed	(51)
S_y	.....	Specific yield	(60)

Ss'	L^{-1}	Specific storage of confining beds	(37)
Ss	L^{-1}	Specific storage of aquifer	(37)
T	L^2T^{-1}	Transmissivity	(1, 6, 7, 9, 10, 11, 12, 13, 14, 15, 18, 19, 20, 21, 22, 23, 26, 30, 32, 33, 34, 35, 36, 38, 39, 40, 41, 42, 43, 45, 46, 47, 48, 49, 50, 51, 54, 59, 60, 61, 62, 68, 69, 70, 71)
$W(u)$	.....	W (well) function of u	(8, 9, 11, 68)
$W(u)_p$	.....	W (well) function of u for pumped control well	(68)
$W(u)_i$	.....	W (well) function of u for image control well	(68)
$W(u,r/B)$	.....	W (well) function of u,r/B	(32, 38)
Y_0	.....	Zero-order Bessel function of the second kind	(24, 27)
Y_1	.....	First-order Bessel function of the second kind	(24, 27)
b	L	Aquifer thickness	(1, 4, 37)
b	L	Initial saturated thickness of unconfined aquifer	(62, 63, 35, 66)
b'	L	Thickness of confining bed	(34, 36, 40, 43, 44, 46, 48)
b'	L	Thickness of upper confining bed	(51)
b''	L	Thickness of lower confining bed	(51)
dh/dl	.....	Hydraulic gradient	(2, 3)

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h	L	Change in water level in aquifer	(24)
n	L	Effective porosity	(3)
r	L	Radial distance from center of control well	(7, 10, 12, 13, 15, 17, 20, 21, 24, 32, 33, 36, 37, 38, 39, 42, 43, 46, 48, 50, 51, 54, 60, 61, 63)
r_1	L	Distance, radial, 1	(17)
r_2	L	Distance, radial, 2	(17)
r_c	L	Radius of pumping/slug well casing or open hole in the interval where water level changes	(25, 26, 30, 31)
r_i	L	Radial distance from center of image pumping well	(70)
r_p	L	Radial distance from center of a pumping/slug well	(69)
r_w	L	Radius of pumping/slug well screen or open hole	(24, 25, 31)
r_w	L	Effective radius of a pumping well	(71)
s	L	Drawdown of head	(6, 9, 11, 13, 14, 15, 20, 32, 38, 41, 45, 47, 49, 54, 59, 65, 66)
s'	L	Corrected drawdown, equal to $s - (s^2/2b)$	(65)
s'	L	Residual drawdown	(22, 23)
s_D	.....	Dimensionless drawdown equal to $4\pi Ts/Q$	(59)
s_i	L	Drawdown component caused by image pumping well	(67)

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s_o	L	Drawdown in observation well	(67, 68)
s_p	L	Drawdown component caused by pumping control well	(67)
s_w	L	Drawdown in the pumping/slug well	(5)
t	T	Time	(7, 10, 12, 13, 14, 16, 20, 21, 26, 30, 33, 39, 42, 44, 50, 54, 60, 61, 69, 70, 71)
t	T	Time since pumping started	(23)
t'	T	Time since pumping ceased	(23)
t_s	.....	Dimensionless time with respect to S	(55, 56, 61)
t_y	.....	Dimensionless time with respect to S_y	(60)
t_1	T	Time, elapsed, 1	(16)
t_2	T	Time elapsed, 2	(16)
u	.....	$r^2S/4Tt$	(6, 7, 8, 10, 12, 22, 32, 33, 35, 38, 39, 41, 42, 44, 49, 50, 52)
u'	.....	$r^2S/4Tt$	(22)
u	.....	Variable of integration	(24, 27, 29)
$u_0(y)$	.....	Defined in equation (54)	(54, 55)
$u_n(y)$	.....	Defined in equation (55)	(54, 56)
u_p	.....	$r^2S/4Tt$ for pumped control well	(69)
u_i	.....	$r^2S/4Tt$ for image control well	(70)
v	.....	$r/w(K'/Tb')^{1/2}$	(35, 36, 41, 43, 44)

v	L/T	Flux-specified discharge	(2)
$f(,v)$	L/T	Average linear velocity	(3)
x	.....	Independent variable in definition of erfc(x)	(53)
x	.....	$r(K'/Tb')^{1/2}$	(45, 46, 47, 48)
y	.....	Variable of integration	(6, 8, 35, 52, 53, 54, 55, 56, 57, 58)
z	.....	Variable of integration	(32)
a	.....	$r_w^2 S/r_c^2$	(24, 25, 27, 28, 29, 31)
b	.....	Tt/r_c^2	(24, 26, 28, 29, 30)
b	.....	$(r/4b)(K'S_s')^{1/2}$	(37)
b	.....	$K_D r^2/b^2$	(54, 55, 56)
b	.....	$(r/4)((K'S'/b'TS) + (K''S''/b''TS)^{1/2})$	(49, 51, 52)
b	.....	$r^2 K_z/(K_r b^2)$	(63)
γ_0	.....	Root of equation (56)	(55, 57)
γ_n	.....	Root of equation (57)	(56, 58)
Δ	.....	Change in parameter (finite difference)	(14, 15, 18, 19, 23)
Δs_t	L	Change in drawdown over one log cycle of time	(18)
Δs_r	L	Change in drawdown over one log cycle of radial distance	(19)
$\Delta(u)$	.....	Function of u defined in equation	(26),(24, 27, 29)
s	.....	S/S_y	(55, 56, 57, 58)

Appendix C
Glossary of Geohydrologic Concepts and Terms

The following definitions and concepts of geohydrologic terms are principally from Fetter (1994). The book by Fetter, *Applied Hydrogeology* (1994), is also an excellent source for ground water analysis and interpretation.

Anisotropy - Anisotropy is that condition in which significant properties are a function of direction. Anisotropy is common in sedimentary sequences in which hydraulic conductivity perpendicular to the bedding planes is less than the hydraulic conductivity parallel to the bedding.

Aquifer - An aquifer is a saturated geologic unit that has sufficient permeability to transmit water at a substantial rate. An aquifer is commonly defined, in terms of water yielding capacity, as a formation, group of formations, or part of a formation that contains sufficient saturated permeable material to yield significant quantities of water to a well or springs.

Aquifer, leaky - A misnomer, but used here and in aquifer test literature to refer to a confined aquifer that receives leakage from adjacent confining beds when the aquifer is stressed by a pumping well. (See confining bed, leaky.)

Aquitard - See preferred term, confining bed or leaky confining bed.

Artesian - Artesian is synonymous with confined; artesian aquifer is equivalent to confined aquifer. An artesian well is a well deriving its water from an artesian or confined aquifer. The water level in an artesian well stands above the top of the artesian or confined aquifer it penetrates.

Confining bed - A confining bed is a geologic unit with minimal permeability. These beds are not permeable enough to yield significant quantities of water to wells or springs. The permeability of aquifers and confining beds is not precisely defined in a quantitative sense, but a confining bed has distinctly less permeability than the aquifer it confines. Other terms that have been used for beds with minimal permeability include aquitard, aquifuge, and aquiclude.

Confining bed, leaky - A leaky confining bed yields a significant quantity of water to the adjacent aquifer when the aquifer is stressed by a pumping well.

Drawdown - Drawdown is the difference between the static water level and the water level after pumping has begun.

Effective radius - The effective radius of a well is that distance, measured radially from the axis of the well, at which the theoretical drawdown based on the logarithmic head distribution equals the actual drawdown just outside the well screen (Jacob, 1947). From the time intercept of the time drawdown logarithmic plot with the zero drawdown line, the effective radius of the pumping well can be determined by the following equation from Jacob (1947, equation 25, p. 1059):

$$r_w^3 = 2.25 \backslash f(Tt,S)$$

Equilibrium, state of - See flow, steady.

Flow, steady - Steady flow occurs when, at any point, the magnitude and direction of the specific discharge are constant in time.

Flow, unsteady - Unsteady or nonsteady flow occurs when, at any point, the magnitude or direction of the specific discharge changes with time.

Ground water, confined - Confined or artesian ground water is under pressure significantly greater than atmospheric, and its upper boundary is the bottom of a bed of distinctly lower hydraulic conductivity than that of the bed in which the confined water occurs.

Head, total - The total head of a liquid at a given point is the sum of three components: (1) elevation head, which is equal to the elevation of the point above a datum; (2) pressure head, which is the height of a column of water that can be supported by the static pressure at the point; and (3) the velocity head, which is the height the kinetic energy of the liquid is capable of lifting the liquid.

Homogeneity - Homogeneity is synonymous with uniformity. A material is homogeneous if its hydrologic properties are identical everywhere. Although no known aquifer or confining bed is homogeneous in detail, models based on the assumption of homogeneity have been determined empirically to be valuable tools for predicting the approximate relation between ground-water flow and hydraulic head in many flow systems.

Hydraulic conductivity - The hydraulic conductivity of a medium is the volume of water at the existing kinematic viscosity and density that will move in unit time under unit hydraulic gradient through a unit area measured at right angles to the direction of flow. Hydraulic conductivity has dimensions of velocity.

Isotropy - Isotropy is that condition in which all significant properties are independent of direction.

Nonequilibrium, state of - See flow, unsteady.

Observation well - An observation well is open to the aquifer throughout a given vertical distance. The water level in an observation well reflects the average head in the aquifer profile that is occupied by screen or perforated casing (Hantush, 1961a).

Permeability, intrinsic - Intrinsic permeability is a measure of the relative ease with which a porous medium can transmit a fluid under a potential gradient. It is a property of the medium alone and is theoretically independent of the nature of the fluid and of the force field causing movement.

Piezometer - A piezometer is a small-diameter pipe open to the aquifer only at its lower end (Hantush, 1961a).

Porosity, effective - Effective porosity is the amount of interconnected pore space available for fluid transmission.

Potentiometric surface - The potentiometric surface at a point is defined by the level to which water will rise in a tightly cased well or piezometer.

Pumping/slug well - The pumping/slug well of an aquifer test is the well through which the aquifer is stressed, for example, by pumping, injection, or change of head.

Saturated zone - The saturated zone is a zone beneath the ground surface in which all voids, large and small, are ideally filled with water under pressure greater than atmospheric.

Specific capacity - The specific capacity of a well is the discharge per unit drawdown. The specific capacity usually decreases both with time and discharge.

Specific discharge - Specific discharge is the rate of discharge of ground water per unit area of porous medium measured at right angles to the direction of flow.

Specific storage - The specific storage of a confined aquifer is the volume of water released from or taken into storage per unit volume of the porous medium per unit change in head.

Specific yield - The specific yield of a rock is the ratio of the volume of water that the saturated rock will yield by gravity to the volume of the rock. Specific yield is determined by tests of unconfined aquifers and is the change that occurs in the volume of water in storage per unit area of unconfined aquifer as the result of a unit change in head. Such a change in storage is produced by the draining or filling of pore space and is, therefore, dependent on particle size, rate of change of the water table, time, and other variables.

Storage coefficient - The storage coefficient is the volume of water an aquifer releases or takes into storage per unit surface area of the aquifer per unit change in head. In a confined aquifer, the water derived from storage with decline in head comes from expansion of the water and compression of the aquifer. In an unconfined aquifer, the volume of water derived from or added to the aquifer by these processes is much smaller compared to that involved in gravity drainage.

Storativity - Synonymous with storage coefficient.

Transmissivity - Transmissivity is the rate at which water of prevailing kinematic viscosity is transmitted through a unit width of the aquifer under a unit hydraulic gradient.

Unsaturated zone - The unsaturated zone is the zone in which water is under less than atmospheric pressure. This zone is also referred to as the vadose zone and the zone of aeration.

Vadose zone - See preferred term, unsaturated zone.

Water table - The water table is that surface in an unconfined aquifer at which the water pressure is atmospheric. It is defined by the level at which water stands in a well that penetrates the aquifer just far enough to hold standing water.

Well loss - A component of drawdown in a discharging well. Well loss is the loss of head in a pumping well due to turbulent flow that accompanies the flow of water through the aquifer, screen, and upward inside the casing to the pump intake.

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This guide analyzes the field methods involved in conducting a geohydrologic analysis, including pretest water level monitoring, pumping phase, and recovery phase. Selected methods of analytical analysis are reviewed with reference to the geohydrologic setting, the stress placed on the aquifer by the pumping well, the observation of aquifer response, the mathematical solution to the hydraulic head response in the aquifer, and the technique for calculating the hydraulic properties of the aquifer. Type curves are included for selected aquifer test methods.

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